Journal of the Operations Research Society of Japan
Vol. 58, No. 3, July 2015, pp. 247–270

SPATIAL INTERACTION MODEL FOR TRIP-CHAINING BEHAVIOR WITH A FOCUS ON CALCULATION EFFICIENCY

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(Received October 7, 2013; Revised March 18, 2015)

Abstract This paper provides a general framework for a spatial interaction model from the viewpoint of "trip-chain" comprising several trips. To estimate the traffic volume distribution of trip-chains, we proposed a new spatial interaction model based on the entropy maximizing model in 2010. In this paper firstly, an efficient calculation method, based on the spatial interaction model, for the proposed trip-chain is discussed. Furthermore, through these mathematical developments, the mathematical relationships between the entropy model, Markov model, and the discrete choice model, which produce the same traffic volume distribution of trip-chains, are clarified. These discussions not only support the entropy model proposed in a previous paper by human sciences based on expected-utility theory but also cover the shortcomings of the existing Markov model and the discrete choice model. It is often pointed out that the Markov model is a pure stochastic model and there is no support from the individual behavior principle. Moreover, the discrete choice model has the problem that the alternative set becomes huge as a result of dealing with trip-chaining behavior, which has a high degree of freedom. We show, under certain assumptions, the Markov model with individual behavior principle and the discrete choice model without enumerating the alternative set. In addition, we clarify the characteristics between the sequential decision making (Markov model) and the simultaneous decision making (discrete choice model) in terms of trip-chaining behavior.

Keywords: Transportation, spatial interaction model, trip-chain, entropy model, Markov model, discrete choice model

1. Introduction

It is apparent from our everyday lives that the flow of people and things is a fundamental element of creating a city. Some examples include the flow of people commuting to work and schools, movement of products, mail, all types of information etc. Thus, attempts have long been made to describe movements (flow) generated between spaces by using concise mathematical formulae.

Wilson's entropy model [33, 34] is an example that plays an important role in research relating to "spatial interactions". Wilson suggests a spatial interaction model based on "the most realizable state" that depends on the concept of maximizing entropy. It gives theoretical roots to the gravity model that is used analogously in physics. Subsequently, entropy models have come to be applied to a wide range of issues such as prior probability and relaxing constraint conditions [25]. Currently, a great number of spatial interaction models exist other than the one proposed and analyzed herein, but many of these are well served by the birth of the entropy model.

Traditional spatial interaction models have been developed to estimate simple movement from origin to destination. However, people frequently visit several destinations during travel and make a sequence of movements, e.g. "comparing several boutiques to buy clothes" and "visiting multiple sightseeing areas in a journey". In the field of traffic engineering, each movement from a place to another place is regarded as a "*trip*," and these sequences of movements are classified as a "*trip-chain*." One of the features of a trip-chain is that each trip that makes up a behavior would have reciprocal relation.

From the standpoint stated above, an entropy-maximizing model for trip-chaining behavior was proposed in 2010 [5]. This entropy model can be regarded as a general extension of Wilson's entropy model. Some researchers also approached trip-chaining behaviors using the entropy maximizing method [19, 26, 32]. Unfortunately, these researches have some restrictions in their formulations, which are difficult to interpret as generalizations of the traditional entropy model. For example, both Tomlinson's and Mazurkiewicz's approaches [19, 32] are limited by probable trip-chaining behaviors. In Roy *et al.* [26] by contrast, a variety of behaviors are considered, but there is no constraint for destination (i.e. the number of visitings).

However, the previous model [5] had problems concerning calculation efficiency. Specifically, estimating parameters needed a long time because the alternative set of trip-chaining behavior became vast. To solve this problem, in this research, **an efficient calculation method for the proposed spatial interaction model for trip-chaining behavior** is discussed. Specifically, it is shown that when the prior probability and transport cost satisfy certain conditions, the derivation of the total urban-transport cost and the adjustment coefficient of each zone reduces to an inverse-matrix calculation. This is focused on the Markov property of a trip-chain, and invokes Akamatsu's idea [2] of Markov analysis of the chosen route model.

Trip-chaining behaviors have been discussed in a number of research papers. One common method to analyze them is the Markovian approach. In particular, the series of research works by Sasaki [27, 28, 30] is highly important, because the techniques to apply the Markov model to a trip-chain are summarized in detail. Additionally, a variety of other models based on the Markovian approach have also been proposed [8, 9, 12, 14, 17, 18]. Meanwhile, various approaches besides the Markovian approach have also been developed including the discrete choice model. These approaches focus on the formulation of the utility and alternatives of trip-chaining behavior [1, 4, 6, 7, 16, 20–22, 29, 31].

Therefore, as the second purpose of this study, through mathematical developments for efficient calculation, **the mathematical relationships between our entropy model, the Markov model, and the discrete choice model for the trip-chaining behavior** are clarified. These discussions not only clarify the theoretical rationale of our entropy model but also cover the shortcomings of the existing Markov model and discrete choice model. The Markov model is a pure stochastic model and there is no support from the individual behavior principle. Therefore, the fact that the theoretical basis is weak when applying the Markov model to trip-chaining behavior has been highlighted [15]. Moreover, difficulty arises in the discrete choice model because it deals with behavior with a high degree of freedom (such as trip-chaining behavior): the alternative set becomes vast [13]. Based on the research presented herein, we show, under certain assumptions, the Markov model with individual behavior principle and the discrete choice model without enumerating the alternative set.

Furthermore, when expressing the trip-chains with the Markov model, in general: (i) selection probability is provided for the first place of visit in the initial conditions, and (ii) travel behavior to the next destination is expressed by a transition probability matrix. In other words, by repeatedly multiplying together the transition probability matrices from (ii), the transition from a particular destination zone to the next destination zone is described sequentially and expresses the chain-type movements. As is also clear from this, the decision making process supposed by the Markov model can be said to be sequential. So it can be thought that initially, people decide their initial place of visit, then once they reach it, decide on their next behavior and once they arrive to their destination, decide on their next behavior again, and so on and so forth.

On the other hand, in the discrete choice model it is supposed that a particular trip-chain will be selected from a range of potential trip-chains (alternative set) based on the utility coefficient. In this case, the decision making process assumed by the model is thought to be simultaneous. In other words, people decide before they leave home (origin), anticipating all of the series of travel behaviors.

In this way, the assumed decision making process is different in the two and, hence, the traffic volume distribution achieved as a result should also be different. Nevertheless, in this research, a Markov model, which produces the same traffic volume distribution of trip-chains with the discrete choice model, will be proposed. This "paradox" indicates not only the characteristics of both models but also the true nature of the decision making of trip-chaining behavior.

This paper is structured as follows: In Section 2, the method based on the entropy-maximizing concept to estimate trip-chaining behavior is summarized. In the discussion, the doubly-constrained(origin-destination constrained) entropy model and the origin-constrained entropy model are shown. This origin-constrained model becomes an important idea for the next section. In Section 3, the derivation from the discrete choice model is also discussed. Even in relation to the trip-chaining behavior, this derivation shows the theoretical basis of the entropy model from the point of view of the individual behavior principle. In Section 4 it is shown that the parameters A_i, B_j , and γ in the doubly-constrained entropy model for trip-chains can be efficiently calculated under certain assumptions. Moreover, in Section 5 the relationship between the entropy model, the Markov model, and the discrete choice model is discussed. There is the assumption of Markovian property when we propose the efficient calculation method. As can be understood from this, there is a strong link between the entropy model and the Markov model for the trip-chaining behavior. Hence in Section 5, the elegant characteristics of the trip-chaining behavior in terms of our entropy model, as well as the discrete choice model and the Markov model, is considered.

2. Entropy Model for Trip-Chaining Behavior

The entropy model for trip-chaining behavior proposed in [5] is summarized in this section.

2.1. Definition of a trip-chain

In this study, we define a *trip-chain* as a sequence of movements which

(i) starts from an origin zone (indexed by i),

(ii) visits several destination zones (indexed by j) successively, and

(iii) goes back to the same origin zone.

Suppose that i and j are the index origin zone and destination zone, respectively, and define that a trip-chain $i\boldsymbol{j}$ is a series of $\Lambda + 1$ trips:

$$\text{origin zone } i \to \text{destination zone } j_1 \to \cdots \to \text{destination zone } j_\Lambda \to \text{origin zone } i, \qquad (1)$$

where $\boldsymbol{j} = [j_1, j_2, \cdots, j_\Lambda]$ is a Λ-dimensional vector. In addition, assume that $t_{i\boldsymbol{j}}$ is the number of people who make a trip-chain $i\boldsymbol{j}$.

Hence, $\boldsymbol{j}$ is interpreted as the "visiting path", and Λ is different for each trip-chain. Some examples are shown in Figure 1.

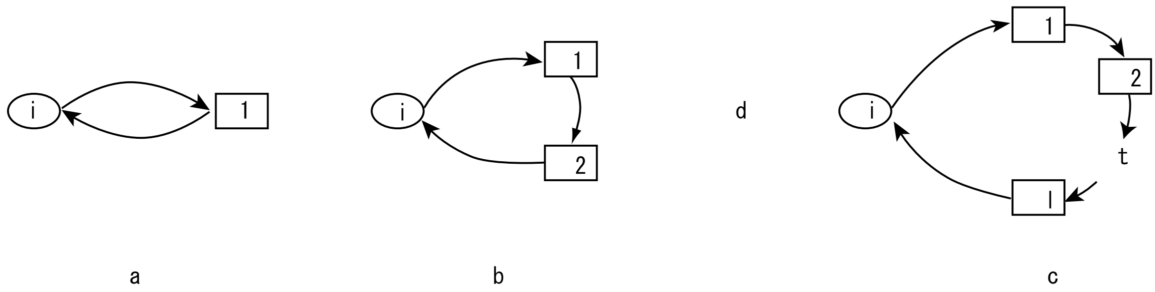

Fig. 1: Examples of trip-chains

2.2. Formulation of doubly-constrained entropy model

Let i and j be index origin zones and destination zones, respectively ($i \in \{1, 2, \cdots, I\}$, $j \in \{1, 2, \cdots, J\}$). In addition, $i\boldsymbol{j}$ is a trip-chain defined by (1). Furthermore, let $t_{i\boldsymbol{j}}$ be the number of individuals who make trip-chain $i\boldsymbol{j}$. For simplicity, it is assumed that the number of destinations in one trip-chain is less than L ($1 \leq \Lambda \leq L$). Here, L is the upper limit of the total zones visited, and is constant in the model.

The main objective is to estimate $t_{i\boldsymbol{j}}$ for all trip-chain paths based on the entropy maximizing method. For this purpose, the following origin-destination constraints of $t_{i\boldsymbol{j}}$ are supposed:

$$O_i = \sum_{\boldsymbol{j} \in \Phi} t_{i\boldsymbol{j}} \qquad (i \in \{1, 2, \cdots, I\}), \tag{2}$$

$$D_j = \sum_{l=1}^{L} \sum_{i=1}^{I} \sum_{\{\boldsymbol{j} \in \Phi | j_l = j\}} t_{i\boldsymbol{j}} \qquad (j \in \{1, 2, \cdots, J\}), \tag{3}$$

where

$$\Phi \stackrel{\text{def}}{=} [\text{individual's alternative set of } \boldsymbol{j}]. \tag{4}$$

In this study, it is assumed that the alternative set is all the trip-chaining behavior possible. For example, in the case of $J = 2, L = 2$, Φ is

$$\Phi = \Big\{ [1],\ [2],\ [1,1],\ [1,2],\ [2,1],\ [2,2] \Big\}. \tag{5}$$

Furthermore, we express the total number of trip-chains as T:

$$T = \sum_{i=1}^{I} \sum_{\{\boldsymbol{j} \in \Phi\}} t_{i\boldsymbol{j}} \left(= \sum_{i=1}^{I} O_i \leq \sum_{j=1}^{J} D_j \right). \tag{6}$$

Besides the origin-destination constraints, we also assume that $t_{i\boldsymbol{j}}$ satisfies the total-transport-cost constraint:

$$C = \sum_{i=1}^{I} \sum_{\boldsymbol{j} \in \Phi} t_{i\boldsymbol{j}} c_{i\boldsymbol{j}}, \tag{7}$$

where $c_{\boldsymbol{ij}}$ is the travel cost of trip-chain $i\boldsymbol{j}$ per individual. $\sum_{i=1}^{I}\sum_{\boldsymbol{j}\in\Phi}$ includes all origin zones and the visiting paths, and thus considers all trip-chain paths in the model.

Now, we derive probability $w\left(\{t_{i\boldsymbol{j}}\}\right)$ to find the distribution of trip-chains $\{t_{i\boldsymbol{j}}\}$. For this purpose, we first assume that $p_{i\boldsymbol{j}}$ is the prior probability of trip-chain $i\boldsymbol{j}$, where

$$\sum_{i=1}^{I}\sum_{\boldsymbol{j}\in\Phi} p_{i\boldsymbol{j}} = 1. \tag{8}$$

Using $p_{i\boldsymbol{j}}$, we can now derive the probability $w\left(\{t_{i\boldsymbol{j}}\}\right)$ that obtains a distribution of trip-chains $\{t_{i\boldsymbol{j}}\}$ as follows:

$$w\left(\{t_{i\boldsymbol{j}}\}\right) = \frac{T!}{\prod_{i=1}^{I}\prod_{\boldsymbol{j}\in\Phi} t_{i\boldsymbol{j}}!}\prod_{i=1}^{I}\prod_{\boldsymbol{j}\in\Phi}\left(p_{i\boldsymbol{j}}\right)^{t_{i\boldsymbol{j}}}. \tag{9}$$

As in the traditional doubly-constrained model, let us maximize probability (9) subject to constraints (2), (3), and (7). Note that it is more convenient to maximize $\ln w\left(\{t_{i\boldsymbol{j}}\}\right)$ rather than $w\left(\{t_{i\boldsymbol{j}}\}\right)$ itself (this transition has no effect because a logarithmic function is a monotonically increasing function). By using Stirling's approximation $N! = N\ln N - N$, the Lagrangian function for this optimization problem is given by:

$$\begin{aligned}
\mathfrak{L}\left(\{t_{i\boldsymbol{j}}\};\lambda,\mu,\gamma\right) = \ln T! &- \sum_{i=1}^{I}\sum_{\boldsymbol{j}\in\Phi}\left(t_{i\boldsymbol{j}}\ln t_{i\boldsymbol{j}} - t_{i\boldsymbol{j}}\right) + \sum_{i=1}^{I}\sum_{\boldsymbol{j}\in\Phi}\left(t_{i\boldsymbol{j}}\ln p_{i\boldsymbol{j}}\right)\\
&+ \sum_{i=1}^{I}\lambda_i\left(O_i - \sum_{\boldsymbol{j}\in\Phi} t_{i\boldsymbol{j}}\right)\\
&+ \sum_{j=1}^{J}\mu_j\left(D_j - \sum_{l=1}^{L}\sum_{i=1}^{I}\sum_{\{\boldsymbol{j}\in\Phi|j_l=j\}} t_{i\boldsymbol{j}}\right)\\
&+ \gamma\left(C - \sum_{i=1}^{I}\sum_{\boldsymbol{j}\in\Phi} t_{i\boldsymbol{j}}c_{i\boldsymbol{j}}\right),
\end{aligned} \tag{10}$$

where λ_i, μ_j, and γ are Lagrange multipliers. The $t_{i\boldsymbol{j}}$'s which maximize $\mathfrak{L}$ and, therefore, constitute the most probable distribution of trip-chains, are the solutions of

$$\frac{\partial\mathfrak{L}}{\partial t_{i\boldsymbol{j}}} = -\ln t_{i\boldsymbol{j}} + \ln p_{i\boldsymbol{j}} - \lambda_i - \mu_{j_1} - \cdots - \mu_{j_\Lambda} - \gamma c_{i\boldsymbol{j}} = 0 \tag{11}$$

and constraint equations (2), (3), and (7). By assuming

$$A_i = \frac{\exp\left[-\lambda_i\right]}{O_i}, \tag{12}$$

$$B_j = \frac{\exp\left[-\mu_j\right]}{D_j}, \tag{13}$$

we consequently obtain

$$t_{ij} = A_i O_i \left(\prod_{l=1}^{\Lambda} B_{j_l} D_{j_l} \right) p_{ij} \exp\left[-\gamma c_{ij}\right]. \tag{14}$$

Furthermore, substituting (14) in (2) and (3), we derive the following:

$$A_i = \left\{ \sum_{j\in\Phi} \left(\prod_{l=1}^{\Lambda} B_{j_l} D_{j_l} \right) p_{ij} \exp\left[-\gamma c_{ij}\right] \right\}^{-1}, \tag{15}$$

$$B_j = \left\{ \sum_{l=1}^{L} \sum_{i=1}^{I} \sum_{\{j\in\Phi | j_l = j\}} A_i O_i \left(\prod_{\substack{l^*=1 \\ l^*\neq l}}^{\Lambda} B_{j_{l^*}} D_{j_{l^*}} \right) p_{ij} \exp\left[-\gamma c_{ij}\right] \right\}^{-1}. \tag{16}$$

This is **a doubly-constrained entropy model for trip-chaining behavior**.

2.3. Formulation of origin-constrained entropy model

As in the traditional entropy model, our entropy model for trip-chaining behavior enables the derivation of four model cases by considering with or without origin-destination constraints. In this sub-section, the origin-constrained entropy model is derived.

In particular, the probability of (9) subject to constraints (2) and (7) is maximized. The Lagrangian function for this optimization problem is given by:

$$\begin{aligned}\mathfrak{L}\left(\{t_{ij}\}; \lambda, \gamma\right) = \ln T! &- \sum_{i=1}^{I} \sum_{j\in\Phi} \left(t_{ij} \ln t_{ij} - t_{ij}\right) + \sum_{i=1}^{I} \sum_{j\in\Phi} \left(t_{ij} \ln p_{ij}\right) \\ &+ \sum_{i=1}^{I} \lambda_i \left(O_i - \sum_{j\in\Phi} t_{ij} \right) \\ &\quad + \gamma \left(C - \sum_{i=1}^{I} \sum_{j\in\Phi} t_{ij} c_{ij} \right),\end{aligned} \tag{17}$$

where λ_i and γ are Lagrange multipliers. The t_{ij}'s, which maximize $\mathfrak{L}$, and that, therefore, constitute the most probable distribution of trip-chains, are the solutions of

$$\frac{\partial \mathfrak{L}}{\partial t_{ij}} = -\ln t_{ij} + \ln p_{ij} - \lambda_i - \gamma c_{ij} = 0 \tag{18}$$

and constraint equations (2) and (7).

By assuming (12), as in Section 2.2, we consequently obtain

$$t_{ij} = A_i O_i p_{ij} \exp\left[-\gamma c_{ij}\right], \tag{19}$$

$$A_i = \left\{ \sum_{j\in\Phi} p_{ij} \exp\left[-\gamma c_{ij}\right] \right\}^{-1}. \tag{20}$$

This is **an origin-constrained entropy model for trip-chaining behavior**.

2.4. Estimation procedure

A procedure to estimate $t_{\boldsymbol{ij}}$ under constraints (2), (3), and (7) is undertaken. The objective is to estimate the spatial interaction, which considers the trip-chain, under the situation that the number of trip from origin zones O_i, the number of stops in destination zones D_j, and total-transport-cost in the region are known.

First, substituting (14) in (7), gives

$$f\left(\gamma\right)=\sum_{i=1}^{I}\sum_{\boldsymbol{j}\in\Phi}A_iO_i\left(\prod_{l=1}^{\Lambda}B_{j_l}D_{j_l}\right)p_{i\boldsymbol{j}}\exp\left[-\gamma c_{i\boldsymbol{j}}\right]c_{i\boldsymbol{j}}-C=0. \tag{21}$$

Using the preceding equations (15), (16), and (21), a calibration method that estimates $I+J+1$ parameters, namely A_i, B_j, and γ is made.

The algorithm proposed is as follows:

i) Starting values, $\gamma=\gamma^0$, $B_j=B_j^0\,(j\in\{1,2,\ldots,J\})$, and $\xi=0$ are set.

ii) Calculate:
$$A_i^{\xi+1}=\left[\sum_{\boldsymbol{j}\in\Phi}\left\{\left(\prod_{l=1}^{\Lambda}B_{j_l}^{\xi}D_{j_l}\right)p_{i\boldsymbol{j}}\exp\left[-\gamma c_{i\boldsymbol{j}}\right]\right\}\right]^{-1}\quad(i\in\{1,2,\ldots,I\})$$
(from (15)).
Then, calculate
$$B_j^{\xi+1}=\left\{\sum_{i=1}^{I}\sum_{\{\boldsymbol{j}\in\Phi|j_l=j\}}A_i^{\xi+1}O_i\left(\prod_{\substack{l^*=1\\l^*\neq l}}^{\Lambda}B_{j_{l^*}}^{\xi}D_{j_{l^*}}\right)p_{i\boldsymbol{j}}\exp\left[-\gamma c_{i\boldsymbol{j}}\right]\right\}^{-1}\quad(j\in\{1,2,\ldots,J\})$$
(from (16)).

iii) If $\left|A_i^{\xi+1}-A_i^{\xi}\right|<\varepsilon_A\,(i\in\{1,2,\ldots,I\})$ and $\left|B_k^{\xi+1}\approx B_j^{\xi}\right|<\varepsilon_B\,(j\in\{1,2,\ldots,J\})$ are satisfied, where ε_A and ε_B are small positive numbers. Then go to iv). If not, set $\xi=\kappa+1$ and go back to ii).

iv) Set $x^0=\gamma^{\kappa}$ and calculate:
$$x^{\kappa+1}=x^{\kappa}-f\left(x^{\kappa}\right)/f'\left(x^{\kappa}\right)$$
iteratively until $\left|x^{\kappa'+1}-x^{\kappa'}\right|<\varepsilon_G$ is satisfied, where ε_G is a small positive number. Then, set $\gamma^{\xi+1}=x^{\kappa'+1}$.

v) Set $\xi=\xi+1$ and go to ii).

Here, first derivative $f'\left(x\right)$ in iv) is

$$f'\left(x\right)=-\sum_{i=1}^{I}\sum_{\boldsymbol{j}\in\Phi}A_iO_i\left(\prod_{l=1}^{\Lambda}B_{j_l}D_{j_l}\right)p_{i\boldsymbol{j}}\exp\left[-xc_{i\boldsymbol{j}}\right]{c_{i\boldsymbol{j}}}^2. \tag{22}$$

3. Derivation of the Entropy Model from the Discrete Choice Model

In this section, we derive the entropy model for the trip-chain by using the multinomial logit model, which is the most popular model of discrete choice model. Even in relation to the trip-chain, this derivation shows the theoretical basis of the entropy model from the point of view of the individual behavior principle.

3.1. Formulation

Consider carrying out a trip-chain $i\boldsymbol{j}$ in which an individual residing in origin zone i repeatedly visits destination zones $\boldsymbol{j} = [j_1, j_2, \cdots, j_\Lambda]$. Suppose that the individuals in origin zone i select the trip-chain $i\boldsymbol{j}$, that maximizes their utilities, from the alternative set Φ for the zones $\boldsymbol{j}$. The utility $U_{\boldsymbol{j}|i}$ when an individual currently residing in origin zone i successively visits the destination zone $\boldsymbol{j}$ is

$$U_{\boldsymbol{j}|i} = V_{\boldsymbol{j}|i} + \varepsilon. \tag{23}$$

Here, $V_{\boldsymbol{j}|i}$ is a fixed utility term and ε is a stochastic variable with a Gumbel distribution with scale parameter η. By assuming this, (23) becomes the definition of utility in the multinomial logit model, so the probability $P_{\boldsymbol{j}|i}$ that an individual in origin zone i selects the trip-chain $i\boldsymbol{j}$ becomes

$$P_{\boldsymbol{j}|i} = \frac{\exp\left[\eta V_{\boldsymbol{j}|i}\right]}{\displaystyle\sum_{\boldsymbol{j}\in\Phi} \exp\left[\eta V_{\boldsymbol{j}|i}\right]}. \tag{24}$$

Thus, if the number of individuals in origin zone i was O_i, the number of individuals $t_{i\boldsymbol{j}}$ conducting the trip-chain $i\boldsymbol{j}$ can be calculated as follows:

$$t_{i\boldsymbol{j}} = O_i \cdot P_{\boldsymbol{j}|i} = O_i \frac{\exp\left[\eta V_{\boldsymbol{j}|i}\right]}{\displaystyle\sum_{\boldsymbol{j}\in\Phi} \exp\left[\eta V_{\boldsymbol{j}|i}\right]}. \tag{25}$$

3.2. Derivation of origin-constrained model

The following assumption is made to obtain a specific function format for the fixed term $V_{\boldsymbol{j}|i}$:

$$V_{\boldsymbol{j}|i} = V'_{\boldsymbol{j}|i} - b\, c_{i\boldsymbol{j}}. \tag{26}$$

Equation (26) breaks down utility into two terms: the term $c_{i\boldsymbol{j}}$ relating to the transport cost and the term $V'_{\boldsymbol{j}|i}$ determined by other remaining factors. Moreover, in the context of this research, utility varies inversely with the transport cost $c_{i\boldsymbol{j}}$ of a direct trip.

If (26) is substituted into (25),

$$t_{i\boldsymbol{j}} = O_i \frac{\exp\left[\eta V'_{\boldsymbol{j}|i}\right] \exp\left[-\gamma c_{i\boldsymbol{j}}\right]}{\displaystyle\sum_{\boldsymbol{j}\in\Phi} \exp\left[\eta V'_{\boldsymbol{j}|i}\right] \exp\left[-\gamma c_{i\boldsymbol{j}}\right]} \tag{27}$$

is obtained($\gamma = b\eta$).

Here, if

$$A_i = \left\{ \sum_{\boldsymbol{j}\in\Phi} \exp\left[\eta V'_{\boldsymbol{j}|i}\right] \exp\left[-\gamma c_{i\boldsymbol{j}}\right] \right\}^{-1}, \tag{28}$$

then t_{ij} can be rewritten in the following concise format:

$$t_{ij} = A_i O_i \exp\left[\eta V'_{\boldsymbol{j}|i}\right] \exp\left[-\gamma c_{\boldsymbol{ij}}\right]. \tag{29}$$

If $\exp\left[\eta V'_{\boldsymbol{j}|i}\right]$ is considered to be equivalent to prior probability, (29) is the origin-constrained entropy model (19) for the trip-chain. More specifically, a relationship between $p_{\boldsymbol{ij}}$ and $\exp\left[\eta V'_{\boldsymbol{j}|i}\right]$ is required such that

$$p_{\boldsymbol{ij}} \propto \exp\left[\eta V'_{\boldsymbol{j}|i}\right]. \tag{30}$$

In other words, even the entropy model for a trip-chain is supported by human sciences based on expected-utility theory.

Furthermore, we suppose the following specific format for utility's fixed term $V'_{\boldsymbol{j}|i}$:

$$V'_{\boldsymbol{ij}} = a \ln S_{j_1} + a \ln S_{j_2} + \cdots + a \ln S_{j_\Lambda} - \Lambda h + \ln \delta_{\boldsymbol{ij}}. \tag{31}$$

The meaning of (31) is that the utility of each visited zone should increase. In (31), after setting the attractiveness of zone j to be S_j, this increase is given by the linear sum of their logarithms. The idea is similar to that of the Huff model [10] for direct trips. In addition, for the trip-chain, we consider the following: First, the reduction in the utility of opportunity cost. Basically, the utility increases with an increase in the number of zones visited because of the increase in attractiveness, thus, it is possible that a selection be made such that visits continue infinitely. Common sense, however, tells us that this type of thing cannot occur because individuals consider the tradeoff of opportunity cost lost through a visit and the attractiveness of the zone. Thus, in (31), the opportunity cost lost through one visit to the zone is h, and it was supposed that $-\Lambda h$ and utility decrease linearly in response to the visits to the destination zones. Finally, we consider $\ln \delta_{\boldsymbol{ij}}$. No matter how attractive a trip-chain is, if the trip-chain is not recognized by an individual it will never be selected. Thus, $\delta_{\boldsymbol{ij}}$ is used to express the existence of recognition. Noting that $\delta_{\boldsymbol{ij}}$ spans the range 0-1, $\ln \delta_{\boldsymbol{ij}}$ is

$$\ln \delta_{\boldsymbol{ij}} = \begin{cases} 0 & \text{(when trip-chain } \boldsymbol{ij} \text{ is recognized)} \\ -\infty & \text{(when trip-chain } \boldsymbol{ij} \text{ is not recognized)} \end{cases}. \tag{32}$$

If this is added to the utility, for the unrecognized route, its utility becomes $-\infty$ and it will not be selected.

If (31) is substituted into (25),

$$t_{\boldsymbol{ij}} = \delta_{\boldsymbol{ij}} A_i O_i \left(\prod_{l=1}^{\Lambda} S_{j_l}^{\alpha} \eta' \right) \exp\left[-\gamma c_{\boldsymbol{ij}}\right] \tag{33}$$

is obtained($\alpha = a\eta$, $\gamma = b\eta$, $\eta' = \exp[-\eta h]$), which is equivalent to the case when the prior probability in the origin-constrained entropy model (19) is assumed to be

$$p_{\boldsymbol{ij}} \propto \delta_{\boldsymbol{ij}} \left(\prod_{l=1}^{\Lambda} S_{j_l}^{\alpha} \eta' \right). \tag{34}$$

3.3. Derivation of doubly-constrained model

Within the traditional spatial interaction model, the doubly-constrained entropy model was derived by the discrete choice model [3]. By generalizing this argument, it is possible to show that the doubly-constrained entropy model could be derived by the discrete choice model, even for a trip-chain in this study.

Now suppose that D_j gives the total number of visitors at destination zone j. At this point the following relationship must hold between $S_{j=1}^{J}$ and D_j:

$$\begin{aligned} D_j &= \sum_{l=1}^{L}\sum_{i=1}^{I}\sum_{\{\boldsymbol{j}\in\Phi|j_l=j\}} t_{i\boldsymbol{j}} \\ &= S_j^{\alpha}\eta' \sum_{l=1}^{L}\sum_{i=1}^{I}\sum_{\{\boldsymbol{j}\in\Phi|j_l=j\}} \delta_{i\boldsymbol{j}} A_i O_i \left(\prod_{\substack{l^*=1\\ l^*\neq l}}^{\Lambda} S_{j_{l^*}}^{\alpha}\eta'\right) \exp\left[-\gamma c_{i\boldsymbol{j}}\right]. \end{aligned} \tag{35}$$

Therefore, assuming B_j as follows:

$$B_j = \left\{ \sum_{l=1}^{L}\sum_{i=1}^{I}\sum_{\{\boldsymbol{j}\in\Phi|j_l=j\}} \delta_{i\boldsymbol{j}} A_i O_i \left(\prod_{\substack{l^*=1\\ l^*\neq l}}^{\Lambda} S_{j_{l^*}}^{\alpha}\eta'\right) \exp\left[-\gamma c_{i\boldsymbol{j}}\right] \right\}^{-1}, \tag{36}$$

we obtained

$$S_j^{\alpha}\eta' = B_j D_j. \tag{37}$$

Substituting these results into Eqs. (28), (29), and (36) gives

$$t_{i\boldsymbol{j}} = \delta_{i\boldsymbol{j}} A_i O_i \left(\prod_{l=1}^{\Lambda} B_j D_j\right) \exp\left[-\gamma c_{i\boldsymbol{j}}\right], \tag{38}$$

$$A_i = \left\{ \sum_{\boldsymbol{j}\in\Phi} \delta_{i\boldsymbol{j}} \left(\prod_{l=1}^{\Lambda} B_j D_j\right) \exp\left[-\gamma c_{i\boldsymbol{j}}\right] \right\}^{-1}, \tag{39}$$

$$B_j = \left\{ \sum_{l=1}^{L}\sum_{i=1}^{I}\sum_{\{\boldsymbol{j}\in\Phi|j_l=j\}} \delta_{i\boldsymbol{j}} A_i O_i \left(\prod_{\substack{l^*=1\\ l^*\neq l}}^{\Lambda} B_{j_{l^*}} D_{j_{l^*}}\right) \exp\left[-\gamma c_{i\boldsymbol{j}}\right] \right\}^{-1}. \tag{40}$$

In other words, by taking

$$p_{i\boldsymbol{j}} \propto \delta_{i\boldsymbol{j}}, \tag{41}$$

(38)—(40) are equivalent to the doubly-constrained entropy model for the trip-chaining behavior.

4. Efficient Computation of Parameters

In Section 2.4, the sequential computation method that uses the relational equation of the adjustment coefficient was explained. This approach has the problem that, whatever prior probability is supposed, the calculation time becomes long even though the parameters can be determined. To avoid this issue, this section presents a method for efficiently calculating the parameters A_i, B_j, and γ of the doubly-constrained entropy model for trip-chains.

4.1. Assumptions in calculation

First, we explain the requisite conditions for applying the computation method of this study. More specifically, we explain the conditions that must be satisfied by the total number of zones visited, L, the prior probability $p_{i\boldsymbol{j}}$ of the trip-chain, $i\boldsymbol{j}$, and the transport cost, $c_{i\boldsymbol{j}}$.

Regarding the upper limit L of the total zones visited, we assume herein after that $L \to \infty$. By setting this, we may generate trip-chains that cause circuit movements to continue infinitely, but if the assumption discussed later were applied regarding prior probability, the following would be true, so this problem would be avoided:

$$\lim_{\Lambda \to \infty} p_{i\boldsymbol{j}} = 0. \tag{42}$$

The next assumption is made for the prior probability $p_{i\boldsymbol{j}}$ of the trip-chain $i\boldsymbol{j}$. In preparation for this, the following transition probabilities are defined:

$$p_{ij} \overset{\text{def}}{=} [\text{the transition probability from the origin zone } i \text{ to the destination zone } j] \,, \tag{43}$$

$$p_{jj^*} \overset{\text{def}}{=} [\text{the transition probability from the destination zone } j \text{ to the destination zone } j^*] \,, \tag{44}$$

$$p_{ji} \overset{\text{def}}{=} [\text{the transition probability from the destination zone } j \text{ to the origin zone } i] \,. \tag{45}$$

The prior probability $p_{i\boldsymbol{j}}$ for each trip-chain $i\boldsymbol{j}$ is assumed to be a product for each trip:

$$p_{i\boldsymbol{j}} = p_{ij_1} \times \prod_{l=1}^{\Lambda-1} p_{j_l j_{l+1}} \times p_{j_\Lambda i}. \tag{46}$$

The idea of setting the prior probability $p_{i\boldsymbol{j}}$ amounts to assuming the Markov property.

Finally, an assumption is made regarding the travel cost $c_{i\boldsymbol{j}}$ for one trip-chain of the trip-chain $i\boldsymbol{j}$. To this end, we define the following to be the travel cost for each trip:

$$c_{ij} \overset{\text{def}}{=} [\text{transport cost from the origin zone } i \text{ to the destination zone } j] \,, \tag{47}$$

$$c_{jj^*} \overset{\text{def}}{=} [\text{transport cost from the destination zone } j \text{ to the destination zone } j^*] \,, \tag{48}$$

$$c_{ji} \overset{\text{def}}{=} [\text{transport cost from the destination zone } j \text{ to the origin zone } i] \,. \tag{49}$$

In addition, the travel cost $c_{i\boldsymbol{j}}$ of the trip-chain in the following argument is given by the sum of the interzonal transport costs:

$$c_{i\boldsymbol{j}} = c_{ij_1} + \sum_{l=1}^{\Lambda-1} c_{j_l j_{l+1}} + c_{j_\Lambda i}. \tag{50}$$

If the prior probability $p_{i\boldsymbol{j}}$ and transport cost $c_{i\boldsymbol{j}}$ are set as mentioned above, $p_{i\boldsymbol{j}} \exp\left[-\gamma c_{i\boldsymbol{j}}\right]$ can be written as

$$p_{i\boldsymbol{j}} \exp\left[-\gamma c_{i\boldsymbol{j}}\right] = C_{ij_1} \times C_{j_1 j_2} \times \cdots \times C_{j_{\Lambda-1} j_\Lambda} \times C_{j_\Lambda i}, \tag{51}$$

and so can be broken down into a multiplication for each trip, where

$$C_{**} \overset{\text{def}}{=} p_{**} \exp\left[-\gamma c_{**}\right]. \tag{52}$$

This property plays a vital role in deriving the parameters.

4.2. Efficient calculation of A_i

The method for efficiently calculating the defining equation (15) of A_i is explained in this section. The equation for ${A_i}^{-1}$ can be given as follows:

$$
\begin{aligned}
A_i^{-1} &= \sum_{\boldsymbol{j}\in\Phi}\left(\prod_{l=1}^{\Lambda} B_{j_l}D_{j_l}\right)p_{i\boldsymbol{j}}\exp\left[-\gamma c_{i\boldsymbol{j}}\right]\\
&= \sum_{j_1=1}^{J}\underbrace{B_{j_1}D_{j_1}C_{ij_1}C_{j_1 i}}_{i[j_1]}\\
&\quad + \sum_{j_1=1}^{J}\sum_{j_2=1}^{J}\underbrace{B_{j_1}D_{j_1}B_{j_2}D_{j_2}C_{ij_1}C_{j_1j_2}C_{j_2 i}}_{i[j_1j_2]}\\
&\quad + \cdots .
\end{aligned}
\tag{53}
$$

To calculate this efficiently, we define the following sequence $\left\{Y_{ji}^{n}\right\}$:

$$
Y_{ji}^{0} = C_{ji}, \tag{54}
$$

$$
Y_{ji}^{n+1} = \sum_{j^*=1}^{J} B_{j^*}D_{j^*}C_{jj^*}Y_{j^*i}^{n}. \tag{55}
$$

The meaning of $\left\{Y_{ji}^{n}\right\}$ is

$Y_{ji}^{n} \overset{\text{def}}{=}$ [the sum of adjustment-coefficient components B_jD_j at n destinations and C_{**} associated with returning to origin i after n visits to a zone from j, across all possible combinations of n visits]. (56)

By using

$$
G_Y \overset{\text{def}}{=} \begin{pmatrix} B_1D_1C_{11} & B_2D_2C_{12} & \cdots & B_JD_JC_{1J}\\ B_1D_1C_{21} & B_2D_2C_{22} & \cdots & B_JD_JC_{2J}\\ \vdots & & \ddots & \vdots\\ B_1D_1C_{J1} & B_2D_2C_{J2} & \cdots & B_JD_JC_{JJ} \end{pmatrix}, \tag{57}
$$

the following can be expressed based on (55):

$$
\begin{pmatrix} Y_{1i}^{n+1}\\ Y_{2i}^{n+1}\\ \vdots\\ Y_{Ji}^{n+1} \end{pmatrix} = G_Y \begin{pmatrix} Y_{1i}^{n}\\ Y_{2i}^{n}\\ \vdots\\ Y_{Ji}^{n} \end{pmatrix}, \tag{58}
$$

and we find

$$\begin{pmatrix} \sum_{n=0}^{\infty} Y_{1i}^n \\ \sum_{n=0}^{\infty} Y_{2i}^n \\ \vdots \\ \sum_{n=0}^{\infty} Y_{Ji}^n \end{pmatrix} = (I - G_Y)^{-1} \begin{pmatrix} Y_{1i}^0 \\ Y_{2i}^0 \\ \vdots \\ Y_{Ji}^0 \end{pmatrix}. \tag{59}$$

Additionally, (53) can be rearranged as

$$\begin{aligned} A_i^{-1} = &\sum_{j=1}^{J} B_j D_j C_{ij} \underbrace{C_{ji}}_{Y_{ji}^0} \\ &+ \sum_{j=1}^{J} B_j D_j C_{ij} \underbrace{\sum_{j_2=1}^{J} B_{j_2} D_{j_2} C_{jj_2} C_{j_2 i}}_{Y_{ji}^1} \\ &+ \cdots \\ = &\sum_{j=1}^{J} B_j D_j C_{ij} \sum_{n=0}^{\infty} Y_{ji}^n. \end{aligned} \tag{60}$$

Therefore, A_i can be calculated by substituting (59) into (60).

4.3. Efficient calculation of B_j

An efficient method of calculating the defining equation (16) for B_j is now derived. As expected, $B_j{}^{-1}$ is obtained as follows:

$$\begin{aligned} B_j^{-1} = &\sum_{l=1}^{\infty} \sum_{i=1}^{I} \sum_{\{\boldsymbol{j} \in \Phi | j_l = j\}} A_i O_i \left(\prod_{\substack{l^*=1 \\ l^* \neq l}}^{\Lambda} B_{j_{l^*}} D_{k_{l^*}} \right) p_{i\boldsymbol{j}} \exp\left[-\gamma c_{i\boldsymbol{j}}\right] \\ = &\sum_{i=1}^{I} \underbrace{A_i O_i C_{ij} C_{ji}}_{i[j]} \\ &+ \sum_{i=1}^{I} \sum_{j_2=1}^{J} \underbrace{A_i O_i B_{j_2} D_{j_2} C_{ij} C_{jj_2} C_{j_2 i}}_{i[jj_2]} + \sum_{i=1}^{I} \sum_{j_1=1}^{J} \underbrace{A_i O_i B_{j_1} D_{j_1} C_{ij_1} C_{j_1 j} C_{ji}}_{i[j_1 j]} \\ &+ \cdots . \end{aligned} \tag{61}$$

To calculate this efficiently, in addition to $\left\{Y_{ji}^n\right\}$, the sequence $\left\{X_{ij}^n\right\}$ is defined as follows:

$$X_{ij}^0 = A_i O_i C_{ij}, \tag{62}$$

$$X_{ij}^{n+1} = \frac{\rho}{J} \sum_{j^*=1}^{J} X_{ij^*}^n B_{j^*} D_{j^*} C_{j^* j}. \tag{63}$$

The meaning of X_{ij}^n is

$$\begin{aligned}X_{ij}^n \overset{\text{def}}{=} &[\text{sum of the adjustment-coefficient components } B_jD_j \text{ at the } n \text{ destinations}\\ &\text{and the adjustment-coefficient components } A_iO_i \text{ at the origin zones}\\ &\text{and } C_{**} \text{ associated with the visit to } j,\\ &\text{after } n \text{ visits to a zone from } i, \text{ across all possible combinations of } n \text{ visits}].\end{aligned} \tag{64}$$

Here, assuming

$$G_X = \frac{\rho}{J}\begin{pmatrix} B_1D_1C_{11} & B_1D_1C_{12} & \cdots & B_1D_1C_{1J} \\ B_2D_2C_{21} & B_2D_2C_{22} & \cdots & B_2D_2C_{2J} \\ \vdots & & \ddots & \vdots \\ B_JD_JC_{J1} & B_JD_JC_{J2} & \cdots & B_JD_JC_{JJ} \end{pmatrix} \tag{65}$$

and (63), we can write

$$\begin{pmatrix} X_{i1}^{n+1} \\ X_{i2}^{n+1} \\ \vdots \\ X_{iJ}^{n+1} \end{pmatrix}^{\mathrm{T}} = \begin{pmatrix} X_{i1}^{n} \\ X_{i2}^{n} \\ \vdots \\ X_{iJ}^{n} \end{pmatrix}^{\mathrm{T}} G_X. \tag{66}$$

By using the same argument as in the previous section, we obtain

$$\begin{pmatrix} \sum_{n=0}^{\infty} X_{i1}^{n} \\ \sum_{n=0}^{\infty} X_{i2}^{n} \\ \vdots \\ \sum_{n=0}^{\infty} X_{iJ}^{n} \end{pmatrix}^{\mathrm{T}} = \begin{pmatrix} X_{i1}^{0} \\ X_{i2}^{0} \\ \vdots \\ X_{iJ}^{0} \end{pmatrix}^{\mathrm{T}} (I - G_X)^{-1}. \tag{67}$$

Furthermore (61) can be rewritten as

$$\begin{aligned} B_j^{-1} = &\sum_{i=1}^{I} \underbrace{(A_iO_iC_{ij})}_{X_{ij}^0}\underbrace{(C_{ji})}_{Y_{ji}^0} \\ &+ \sum_{i=1}^{I} \underbrace{(A_iO_iC_{ij})}_{X_{ij}^0}\underbrace{\left(\sum_{j_2=1}^{J} B_{j_2}D_{j_2}C_{jj_2}C_{j_2i}\right)}_{Y_{ji}^1} \\ &+ \sum_{i=1}^{I} \underbrace{\left(\sum_{j_1=1}^{J} A_iO_iB_{j_1}D_{j_1}C_{ij_1}C_{j_1j}\right)}_{X_{ij}^1}\underbrace{(C_{ji})}_{Y_{ji}^0} \\ &+ \sum_{i=1}^{I} X_{ij}^0Y_{ji}^2 + \sum_{i=1}^{I} X_{ij}^1Y_{ji}^1 + \sum_{i=1}^{I} X_{ij}^2Y_{ji}^0 + \cdots \\ = &\sum_{i=1}^{I}\left(\sum_{n=0}^{\infty} X_{ij}^n \times \sum_{n=0}^{\infty} Y_{ji}^n\right). \end{aligned} \tag{68}$$

Hence if (59) and (67) are substituted into (68), B_j^{-1} can be obtained.

4.4. Efficient calculation of C

Finally, the derivation of parameter γ, relating to the total-transport-cost within the city, is explained herein. However, since γ itself does not have a defining equation, the calculation method for the estimated value of the total-transport-cost $\hat{C}(\gamma)$ is given by a certain distance resistance coefficient γ.

In preparation for this, the following three will be derived for cases when the distance resistance coefficient γ has been provided:

$$T_{ij}^{OD}(\gamma) \stackrel{\text{def}}{=} [\text{the number of individuals moving from origin zone } i \text{ to destination zone } j], \tag{69}$$

$$T_{jj^*}^{DD}(\gamma) \stackrel{\text{def}}{=} [\text{the number of individuals moving from destination zone } j \text{ to destination zone } j*], \tag{70}$$

$$T_{ji}^{DO}(\gamma) \stackrel{\text{def}}{=} [\text{the number of individuals returning home from destination zone } j \text{ to origin zone } i]. \tag{71}$$

The number of those moving from origin to destination $i \to j$, $T_{ij}^{OD}(\gamma)$, will be derived. This can be calculated as below by utilizing the aforementioned sequence $\{Y_{ji}^n\}$:

$$\begin{aligned} T_{ij}^{OD}(\gamma) =& t_{i[j]} + \sum_{j_2=1}^{J} t_{i[jj_2]} + \cdots \\ =& A_i O_i B_j D_j C_{ij} \underbrace{C_{ji}}_{Y_{ji}^0} \\ &+ A_i O_i B_j D_j C_{ij} \underbrace{\sum_{j_2=1}^{J} B_{j_2} D_{j_2} C_{jj_2} C_{j_2 i}}_{Y_{ji}^1} \\ &+ \cdots \\ =& A_i O_i B_j D_j C_{ij} \sum_{n=0}^{\infty} Y_{ji}^n. \end{aligned} \tag{72}$$

Next, the number of those touring around $j \to j^*$, $T_{jj^*}^{DD}(\gamma)$ will be derived. This can also be calculated as below by utilizing the sequence $\{X_{ij}^n\}$ and $\{Y_{ji}^n\}$:

$$
\begin{aligned}
T^{DD}_{jj^*}(\gamma) &= \sum_{i=1}^{I} t_{i[jj*]} + \sum_{i=1}^{I}\sum_{j_3=1}^{J} t_{i[jj*j_3]} + \sum_{i=1}^{I}\sum_{j_1=1}^{J} t_{i[j_1jj*]} + \cdots \\
&= \sum_{i=1}^{I}\left\{ \underbrace{(A_iO_iC_{ij})}_{X^0_{ij}} (B_jD_jB_{j^*}D_{j^*}C_{jj^*}) \underbrace{(C_{j^*i})}_{Y^0_{j^*i}} \right\} \\
&\quad + \sum_{i=1}^{I}\left\{ \underbrace{(A_iO_iC_{ij})}_{X^0_{ij}} (B_jD_jB_{j^*}D_{j^*}C_{jj^*}) \underbrace{\left(\sum_{j_3=1}^{J} B_{j_3}D_{j_3}C_{j^*j_3}C_{j_3i}\right)}_{Y^1_{j^*i}} \right\} \\
&\quad + \sum_{i=1}^{I}\left\{ \underbrace{\left(\sum_{j_1=1}^{J} A_iO_iB_{j_1}D_{j_1}C_{ij_1}C_{j_1j}\right)}_{X^1_{ij}} (B_jD_jB_{j^*}D_{j^*}C_{jj^*}) \underbrace{(C_{j^*i})}_{Y^0_{j^*i}} \right\} \\
&\quad + \cdots \\
&= \sum_{i=1}^{I}\left\{ \sum_{n=0}^{\infty} X^n_{ij} \times (B_jD_jB_{j^*}D_{j^*}C_{jj^*}) \times \sum_{n=0}^{\infty} Y^n_{ji} \right\}. \qquad (73)
\end{aligned}
$$

Finally, there is the number of those transporting back home from $j \to i$, $T^{DO}_{ji}(\gamma)$ and this is as follows:

$$
\begin{aligned}
T^{DO}_{ji}(\gamma) =& t_{i[j]} + \sum_{j_1=1}^{J} t_{i[j_1j]} + \cdots \\
=& \underbrace{(A_iO_iC_{ij})}_{X^0_{ij}} (B_jD_jC_{ji}) \\
&+ \underbrace{\left(\sum_{j_1=1}^{J} A_iO_iB_{j_1}D_{j_1}C_{ij_1}C_{j_1j}\right)}_{X^1_{ij}} (B_jD_jC_{ji}) \\
&+ \cdots \\
=& \sum_{n=0}^{\infty} X^n_{ij} B_jD_jC_{ji}. \qquad (74)
\end{aligned}
$$

In this way, if the obtained $T^{OD}_{ij}(\gamma)$, $T^{DD}_{jj^*}(\gamma)$, and $T^{DO}_{ji}(\gamma)$ are each multiplied by the inter-zone movement cost (47), (48), and (49) then the estimated value of the total cost for the whole city $\hat{C}(\gamma)$ can be obtained:

$$\hat{C}(\gamma) = \sum_{i=1}^{I}\sum_{j=1}^{J} T_{ij}^{OD}(\gamma)\, c_{ij} + \sum_{j=1}^{J}\sum_{j^*=1}^{J} T_{jj^*}^{DD}(\gamma)\, c_{jj^*} + \sum_{j=1}^{J}\sum_{i=1}^{I} T_{ji}^{DO}(\gamma)\, c_{ji}. \tag{75}$$

4.5. Method of parameter determination utilizing inverse matrix

The discussions in Section 4.2-4.4 showed that the adjustment coefficients A_i and B_j, and the estimated value of the total-transport-cost, $\hat{C}(\gamma)$, can be calculated efficiently. If these are used, reiteration calculations used to determine the parameters can be conducted faster than by using the defining equations directly. However, in this case, the Newton-Raphson method, which was used when estimating the distance resistance coefficient γ in Section 2.4 iv) cannot be applied. This is because the differential of total cost constraint conditions equivalent to (22) cannot be analyzed for each trip. Therefore, to estimate the distance resistance coefficient γ using (75), an algorithm that doesn't use derivatives needs to be applied.

As one example of this, the procedure for an iterative method using golden section method [11] is shown:

i) Set the initial value as $\gamma = \gamma^0$, $B_j = B_j^0\,(j \in \{1,2,\ldots,J\})$, $\xi = 0$.

ii) Calculate $\displaystyle\sum_{n=0}^{\infty} Y_{ji}^{n}$ $(i \in \{1,2,\ldots,I\}.\ j \in \{1,2,\ldots,J\})$ using B_j^{ξ} ($\leftarrow$ depends on (59)).

Calculate $A_i^{\xi+1} = \left\{\displaystyle\sum_{j=1}^{J} B_j^{\xi} D_j^{\xi} C_{ij} \sum_{n=0}^{\infty} Y_{ji}^{n}\right\}^{-1}$
$(i \in \{1,2,\ldots,I\})$ ($\leftarrow$ depends on (60)).

Calculate $\displaystyle\sum_{n=0}^{\infty} X_{ij}^{n}$ $(i \in \{1,2,\ldots,I\}.\ j \in \{1,2,\ldots,J\})$ using $A_i^{\xi+1}$, B_j^{ξ} ($\leftarrow$ depends on (67)).

Calculate $B_j^{\xi+1} = \left\{\displaystyle\sum_{i=1}^{I}\left(\sum_{n=0}^{\infty} X_{ij}^{n} \times \sum_{n=0}^{\infty} Y_{ji}^{n}\right)\right\}^{-1}$
$(j \in \{1,2,\ldots,J\})$ ($\leftarrow$ depends on (68)).

iii) Go to iv) if $\left|A_i^{\xi+1} - A_i^{\xi}\right| < \varepsilon_A\,(i \in \{1,2,\ldots,I\})$ and $\left|B_j^{\xi+1} - B_j^{\xi}\right| < \varepsilon_B\,(j \in 1,2,\ldots,J\})$, where ε_A and ε_B are small positive numbers. Otherwise go to ii) by setting $\xi = \xi + 1$.

iv) Set $x^0 = \gamma^{\xi}$ and set the counter to be $\kappa = 0$. By applying the golden section method on a positive integer ε_G that is small enough, and stop at the point when:

$$|f(x^{\kappa})| < \varepsilon_G.$$

Let $\gamma^{\xi+1} = x^{\kappa'+1}$.

v) Finish if $\gamma^{\xi+1} = \gamma^{\xi}$. Otherwise set $\xi = \xi + 1$ and go to ii).

Reiterate the above until there is convergence. Furthermore, the function $f(x)$ in the aforementioned iv) is as shown below:

$$f(x) = \sum_{i=1}^{I}\sum_{j=1}^{J} T_{ij}^{OD}(x)\, c_{ij} + \sum_{j=1}^{J}\sum_{j^*=1}^{J} T_{jj^*}^{DD}(x)\, c_{jj^*} + \sum_{j=1}^{J}\sum_{i=1}^{I} T_{ji}^{DO}(x)\, c_{ji} - C. \tag{76}$$

5. Derivation of Markov Model

In the previous section, the efficient calculation method for determining the parameters in trip-chains was discussed. It was shown that the adjustment coefficients such as A_i or B_j return to an inverse matrix calculation but there is the assumption that the prior probability has the Markovian property. As can be understood from this, under the assumptions in Section 4.1, there is a strong link between the Markov model and the entropy model for the trip-chaining behavior. Hence in this section, the relationship between the doubly-constrained entropy model, as well as the discrete choice model and the Markov model for the trip-chains is considered.

5.1. Markov process derived by the doubly-constrained model

Firstly, the relationship between the doubly-constrained entropy model and the Markov model of the trip-chains will be discussed. Specifically, under the assumptions of Section 4.1, the traffic volume distribution estimated by the doubly-constrained entropy model for the trip-chains can also be denoted by the Markov model. In preparation for this, the following two items are calculated:

(i) the proportion of tourists who visit j as the $k+1$th destination out of the tourists who visit $[j_1, \cdots, j_k]$ by the kth visit,

(ii) the proportion of tourists returning without visiting the $k+1$th destination out of the tourists who visit $[j_1, \cdots, j_k]$ by the kth visit.

Firstly, the proportion of tourists that visit j on their $k+1$th destination $r\{j_{k+1} = j | i[j_1, \cdots, j_k]\}$ out of the tourists who visit $[j_1, \cdots, j_k]$ by the kth visit having originated from origin zone i is derived. In other words,

$$r\{j_{k+1} = j | i[j_1, \cdots, j_k]\} \stackrel{\text{def}}{=} \frac{\text{the number of tourists whose path to the } k+1\text{th destination is } i\,[j_1, \cdots, j_k, j]}{\text{the number of tourists whose path to the } k\text{th destination is } i\,[j_1, \cdots, j_k]} \tag{77}$$

should be sought. Here,

$$\begin{aligned}
&[\text{the number of tourists whose path to the } k\text{th destination is } i\,[j_1, \cdots, j_k]] \\
&= t_{i[j_1,\cdots,j_k]} + \sum_{j_{k+1}=1}^{J} t_{i[j_1,\cdots,j_k,j_{k+1}]} + \cdots \\
&= A_i O_i B_{j_1} D_{j_1} \cdots B_{j_k} D_{j_k} C_{ij_1} \cdots C_{j_{k-1}j_k} \left(\underbrace{C_{j_k i}}_{Y^0_{j_k i}} + \underbrace{\sum_{j_{k+1}=1}^{J} B_{j_{k+1}} D_{j_{k+1}} C_{j_k j_{k+1}} C_{j_{k+1} i}}_{Y^1_{j_k i}} + \cdots \right) \\
&= A_i O_i B_{j_1} D_{j_1} \cdots B_{j_k} D_{j_k} C_{ij_1} \cdots C_{j_{k-1}j_k} \sum_{n=0}^{\infty} Y^n_{j_k i}.
\end{aligned} \tag{78}$$

Similarly,

[the number of tourists whose path to the $k+1$th destination is $i\,[j_1, \cdots, j_k, j]$]

$$= A_i O_i B_{j_1} D_{j_1} \cdots B_{j_k} D_{j_k} B_j D_j C_{ij_1} \cdots C_{j_{k-1} j_k} C_{j_k j} \sum_{n=0}^{\infty} Y_{ji}^n. \tag{79}$$

Therefore, $r\{j_{k+1} = j | i[j_1, \cdots, j_k]\}$ can be obtained by substituting (78) and (79) into (77), and thus the following can be obtained:

$$\begin{aligned} & r\{j_{k+1} = j | i[j_1, \cdots, j_k]\} \\ & = \frac{A_i O_i B_{j_1} D_{j_1} \cdots B_{j_k} D_{j_k} B_j D_j C_{ij_1} \cdots C_{j_{k-1} j_k} C_{j_k j} \sum_{n=0}^{\infty} Y_{ji}^n}{A_i O_i B_{j_1} D_{j_1} \cdots B_{j_k} D_{j_k} C_{ij_1} \cdots C_{j_{k-1} j_k} \sum_{n=0}^{\infty} Y_{j_k i}^n} \\ & = B_j D_j C_{j_k j} \frac{\sum_{n=0}^{\infty} Y_{ji}^n}{\sum_{n=0}^{\infty} Y_{j_k i}^n}. \end{aligned} \tag{80}$$

Next, the proportion of tourists returning home without visiting the $k+1$th destination $r\{\text{home}|i[j_1, \cdots, j_k]\}$ out of those tourists who visit $[j_1, \cdots, j_k]$ as a destination up to the kth destination having originated from origin zone i will be derived. In other words,

$$\begin{aligned} & r\{\text{home}|i[j_1, \cdots, j_k]\} \\ & \stackrel{\text{def}}{=} \frac{t_{i[j_1, \cdots, j_k]}}{\text{the number of tourists whose path to the } k \text{ th destination is } i\,[j_1, \cdots, j_k]}. \end{aligned} \tag{81}$$

This, based on (78), becomes

$$\begin{aligned} r\{\text{home}|i[j_1, \cdots, j_k]\} &= \frac{A_i O_i B_{j_1} D_{j_1} \cdots B_{j_k} D_{j_k} C_{ij_1} \cdots C_{j_{k-1} j_k} C_{j_k i}}{A_i O_i B_{j_1} D_{j_1} \cdots B_{j_k} D_{j_k} C_{ij_1} \cdots C_{j_{k-1} j_k} \sum_{n=0}^{\infty} Y_{j_k i}^n} \\ &= \frac{C_{j_k i}}{\sum_{n=0}^{\infty} Y_{j_k i}^n}. \end{aligned} \tag{82}$$

From (80) and (82), under the assumptions in Section 4.1, it can be seen that as a result of t_{ij} being split into factors for each trip, there is no need to depend on the information regarding the path taken before arriving at a particular zone j_k, and hence the proportion of the next trip to take place can be determined without having to depend on the past. This means that the travel behavior can be expressed utilizing the Markov model. Therefore, if the transition probability matrix is defined using (80) and (82), the traffic volume distribution estimated by the entropy model can also be described using the Markov model.

Hence, we will try to describe the traffic volume distribution estimated by the entropy model as an absorbing Markov process based on the aforementioned arguments. What must be noted here is that the specific values of (80) and (82) vary depending on i and so the Markov process has to be denoted for each origin zone.

Now, think of $\mathfrak{S} = \{\mathfrak{S}_{\mathrm{H}}, \mathfrak{S}_1, \cdots, \mathfrak{S}_J\}$ as the state space $\mathfrak{S}$. Here, $\mathfrak{S}_{\mathrm{H}}$ is the state in which a particular tourist (after ending travel) is returning home and $\mathfrak{S}_j$ is the state in which a particular tourist is visiting zone j. Moreover, $\mathfrak{S}_{\mathrm{H}}$ is an absorbing state and $\mathfrak{S}_1, \cdots, \mathfrak{S}_J$ are transient states. Then, of the total number of tourists, O_i, departing origin zone i, the number of those in state $\mathfrak{S}_*$ at nth step is expressed as $\pi_*^i(n)$ and the following vector is defined:

$$\boldsymbol{\pi^i}(n) = \left[\pi_{\mathrm{H}}^i(n), \pi_1^i(n), \cdots, \pi_J^i(n)\right]. \tag{83}$$

In other words, $\pi_j^i(n)$ is the number who are in zone j as the nth destination out of the tourists who originated from origin zone i, and $\pi_{\mathrm{H}}^i(n)$ is the number who have already returned home having visited less than n places out of the tourists who originated from origin zone i. As apparent from the definition, it should be noted that the following is established for an optional n:

$$\pi_{\mathrm{H}}^i(n) + \sum_{j=1}^{J} \pi_j^i(n) = O_i. \tag{84}$$

Now, the initial state of the Markov processis calculated i.e. $\boldsymbol{\pi^i}(1)$. First, $\pi_{\mathrm{H}}^i(1)$ is clearly

$$\begin{aligned}\pi_{\mathrm{H}}^i(1) &= [\text{the number of people who return home at step 1}]\\ &= 0.\end{aligned} \tag{85}$$

Moreover, with regards to $\pi_j^i(1)$, it can be calculated as

$$\begin{aligned}\pi_j^i(1) &= [\text{the number of tourists who depart from zone } i \text{ with the first place of visit as } j]\\ &= T_{ij}^{OD}(\gamma)\\ &= A_i O_i B_j D_j C_{ij} \sum_{n=0}^{\infty} Y_{ji}^n.\end{aligned} \tag{86}$$

In other words, the initial state $\boldsymbol{\pi^i}(1)$ is

$$\boldsymbol{\pi^i}(1) = \left[\underbrace{0}_{\pi_H^i(1)}, \underbrace{A_i O_1 B_1 D_1 C_{i1} \sum_{n=0}^{\infty} Y_{1i}^n}_{\pi_1^i(1)}, \cdots, \underbrace{A_i O_J B_J D_J C_{iJ} \sum_{n=0}^{\infty} Y_{Ji}^n}_{\pi_J^i(1)}\right]. \tag{87}$$

Next, the transition probability matrix P_i will be shown. For this, the following should be applied using (80) and (82):

$$P_i = \begin{array}{c c} & \begin{array}{cccc} \mathrm{H} & 1 & \cdots & J \end{array} \\ \begin{array}{c} \mathrm{H} \\ 1 \\ \vdots \\ J \end{array} & \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \dfrac{C_{1i}}{\sum_{n=0}^{\infty} Y_{1i}^n} & B_1 D_1 C_{11} \dfrac{\sum_{n=0}^{\infty} Y_{1i}^n}{\sum_{n=0}^{\infty} Y_{1i}^n} & & B_J D_J C_{1J} \dfrac{\sum_{n=0}^{\infty} Y_{Ji}^n}{\sum_{n=0}^{\infty} Y_{1i}^n} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{C_{Ji}}{\sum_{n=0}^{\infty} Y_{Ji}^n} & B_1 D_1 C_{J1} \dfrac{\sum_{n=0}^{\infty} Y_{1i}^n}{\sum_{n=0}^{\infty} Y_{Ji}^n} & \cdots & B_J D_J C_{JJ} \dfrac{\sum_{n=0}^{\infty} Y_{Ji}^n}{\sum_{n=0}^{\infty} Y_{Ji}^n} \end{bmatrix} \end{array}. \tag{88}$$

Here, P_i is a transition probability matrix and hence the sum of each line is 1:

$$\frac{C_{ji}}{\sum_{n=0}^{\infty} Y_{ji}^n} + \sum_{j^*=1}^{J} B_{j^*} D_{j^*} C_{jj^*} \frac{\sum_{n=0}^{\infty} Y_{j^*i}^n}{\sum_{n=0}^{\infty} Y_{ji}^n} = 1. \tag{89}$$

It is clear that traffic volume amount derived by the Markov process shown above is equivalent to that estimated by a doubly-constrained entropy model for trip-chains. In other words, it has been made apparent that the entropy model for trip-chains described under the assumptions of Section 4.1 can be described by a Markov model.

5.2. Simultaneous decision making and sequential decision making

Finally, the meaning of (87) and (88) will be considered. For this it would be useful to examine the decision making process supposed by the Markov model and the discrete choice model.

As explained in Introduction, the decision making process supposed by the Markov model can be regarded as sequential. Meanwhile, that assumed by the discrete choice model is regarded as simultaneous. In this way, the assumed decision making process is different in the two models and, hence, the traffic volume distribution achieved in each model should also be different. Nevertheless, in (87) and (88), a Markov model, which produces the same traffic volume distribution of trip-chains with the discrete choice model, was proposed. In many existing research utilizing Markov models, the classical spatial interaction model (for example the Huff model) is often assumed when setting the transition probabilities (for example [23, 24, 28]). Simply put, it is a formulation considering only $S_{j^*}^{\alpha}\eta' \exp[-\gamma c_{jj^*}]$ from (88). Contrastingly, the transition probabilities in this research have the component $\sum_{n=0}^{\infty} {Y''_{j^*i}}^n$ added (the $\sum_{n=0}^{\infty} {Y''_{j^*i}}^n$ in the denominator is an adjustment term to make the row-sum 1 and hence is not essential). In other words, by using the weighting Y_{ji}, the sequential decision making (Markov model) and the simultaneous decision making (discrete choice model) have the same structure. Conversely, if $\sum_{n=0}^{\infty} {Y''_{j^*i}}^n$ is not weighted by the transition probabilities, the traffic volume distribution obtained from the two models will be different.

Moreover, it should be noted that even in the discrete choice model, it was assumed that the "utility function is split into sums for each trip". Only based on this supposition, the movement behavior of people achieves the Markov property. If the above assumptions are not satisfied (for example, the loss of opportunity cost changes non-linearly in respect to the number of visited zones), an equivalent Markov model cannot be constructed.

Although under several constraints stated above, the equivalence of the Markov model and the discrete choice model in terms of the traffic volume distribution of trip-chains has become apparent. Based on this argument, we can cover the several shortcomings of the existing Markov model and the discrete choice model. In the Markov model, (i) the theoretical vulnerability in applying the Markov model to the trip-chains and (ii) the suitability of the sequential decision making process were pointed out as problems. By the discussion in this section, we consider that we could resolve the theoretical vulnerability of (i). Moreover, even (ii) should be resolved by interpreting it as follows. The main idea that the sequential decision making process is not appropriate for trip-chains is based on the fact that the transitions between destination zones are described by a classical spatial interaction model. As a result of being described by information only relating to the destination zone ($S_{j^*}^{\alpha}\eta' \exp[-\gamma c_{jj^*}]$), the danger of the true nature of the trip-chaining behavior being lost was pointed out. On the other hand, in the model proposed in this paper, the factor $\sum_{n=0}^{\infty} {Y''_{j^*i}}^n$ is considered. This factor is similar to the weighting taking into consideration the various motion behaviors in the future. In this case, it is clear that the trip-chaining behavior's characteristic, that is "decision making based on future consideration", is not lost. Hence, under the formulations in this paper, the problems previously raised in respect to the Markov model have been avoided.

Moreover, by the application of the discrete choice model, the discussion in this paper provides essential suggestions to enumerate the alternative set, which is often raised as a difficult issue. If simultaneous decision making is assumed, the number of alternative set becomes huge, since various trip-chaining behaviors are considered, and its calculation becomes complicated. However, this issue is precisely the same structure with the calculation of A_i and B_j. Therefore, under the assumptions used in this research, it is clear that there is no need for complicated calculations and it could result in inverse matrix calculation. By the relationship between the Markov model and the discrete choice model through entropy model presented in this paper, we consider to propose the Markov model with individual behavior principle and the discrete choice model without enumerating the alternative set.

6. Conclusion

This paper provides a general framework for a spatial interaction model from the viewpoint of "trip-chain" comprising several trips. Specifically, the classical entropy model by Wilson et al. [33] has been expanded and a spatial interaction model for trip-chains constructed. In this paper firstly, mathematical discussion was conducted for efficiently calculating the entropy model for trip-chains. In our previous models, there is generally the need to derive the adjustment coefficients for satisfying the origin/destination constraint conditions, but in the previous models for trip-chains, the defining equations for the adjustment coefficients were complicated. To avoid this problem, in this research focus is given to the breaking down of the trip-chains and it has been shown that under certain conditions, the adjustment coefficients resolve to an inverse matrix calculation.

Furthermore, through these mathematical developments, the mathematical relationships between the entropy model, Markov model, and the discrete choice model, which produce the same traffic volume distribution of trip-chains, were clarified. Discussions not only supported the entropy model proposed in previous paper by human sciences based on expected-utility theory but also covered the shortcomings of the existing Markov model and the discrete choice model. It was often pointed out that Markov model was a pure stochastic model and there were no support from individual behavior principle. Moreover, in the discrete choice model the alternative set becomes huge as a result of dealing with trip-chaining behavior, which had a high degree of freedom. This current research shows, under certain assumptions, the Markov model with individual behavior principle and the discrete choice model without enumerating the alternative set. In addition, we clarified the characteristics between the sequential decision making (Markov model) and the simultaneous decision making (discrete choice model) in terms of trip-chaining behavior.

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