Mathematical Analysis of Electric Vehicle Movement With Respect To Multiple Charging Stops

Yudai Honma[1] and Shigeki Toriumi[2]

Abstract: As environmental issues become more prominent, electric vehicles (EV) have attracted an increasing amount of attention. However, the continuous cruising distance of an EV is limited to approximately 160 km, which is insufficient for everyday use. Battery capacity is the limiting factor in long-distance EV travel, because the vehicles need to stop at EV recharging stations multiple times. In Japan, there are more than 2,000 EV charge stations, but there are, at most, two rapid chargers. When multiple users converge at the station, a queuing (or waiting) condition is created, which may lead to a call-loss condition. In other words, an appropriate number of chargers must be installed at each station when planning the EV infrastructure. Therefore, the number of vehicles entering the station must be estimated. In this study, a mathematical model based on the supporting infrastructure for widespread EV use is proposed to estimate the number of vehicles arriving at each charge station. **DOI: 10.1061/(ASCE)EY.1943-7897.0000356.**

Author keywords: Electric vehicle (EV); EV supporting infrastructure; Multiple routes; Multiple stops.

Introduction

Since the start of the 20th century, there has been an enormous amount of fossil fuel usage. It is not an overstatement to say that the current level of prosperity in the world has been supported by this huge consumption of energy (MacKenzie 2000). However, this exponential increase in energy consumption throughout the 20th century and continuing today introduced several problems. Among them, the depletion of fossil fuels is one of the most serious and imminent (Roberts 2004). The depletion of oil, in particular, has been discussed by various organizations and scientists. In the transportation sector, which mainly consists of the automobile industry that is almost entirely dependent on oil, a reduction in CO_2 emissions and a departure from dependence on oil are acute challenges (IEA 2004).

Against this backdrop, electric vehicles (EVs) have gained prominence in recent years. Conceptual EVs have been developed in the past, but several factors have prevented their practical deployment. Among these factors, the major disadvantage was a short driving distance, limited by battery performance and the time required to recharge the battery (Larminie and Lowry 2003). In recent years, however, owing to the increasing prevalence of hybrid cars, the development of batteries such as lithium-ion batteries has led to significant advances in EV performance. However, even in recent models of EVs, the driving distance covered by a car with a fully charged battery is 100–160 km, at most (Nissan Motor Company 2014). Alternatively, it will take approximately 30 min to recharge the battery to approximately 80% capacity using special, rapid-charging equipment (Husain 2010). This distance is very unsatisfactory compared with the 400 km (or more) covered by a gas-engine car with a full tank.

Thus, it is inevitable to develop the EV-support infrastructure for EV users. In fact, there are continuous efforts to popularize the EV-support infrastructures in the United States (Morrow et al. 2008) as well as in Japan (MLIT 2011). Furthermore, numerous models have been proposed to analyze a system of EV-support infrastructure (e.g., Bapna et al. 2002; Kuby et al. 2004; Kuby and Lim 2005, 2007; Kuby et al. 2009; Melaina and Bremson 2008; Upchurch et al. 2009; Lim and Kuby 2010; Upchurch and Kuby 2010 and the references therein). In particular, Kuby and Lim (2005) developed the flow refueling location model to analyze the distribution of EV-charging stations. They also applied this model to real-world networks at both the metropolitan and state scales (Kuby et al. 2004, 2009), and extended it to stations with limited capacities (Upchurch et al. 2009), locations along arcs (Kuby and Lim 2007), and maximizing total trip lengths instead of the total number of trips (Kuby et al. 2009). Similarly, for the Japanese EV infrastructure, Ishigame and Matsuda (2011) also calculated the favorable locations of EV-charging stations in Osaka Prefecture.

However, these flow refueling location models are generally social-optimum models. EVs are allocated to EV stations to socially optimize the above objective indices, and the main concerns of such previous studies are the effective locations of EV stations. In real-world situations, the EV driving route is not feasible and EV users spontaneously determine their driving routes. The model to describe the EV route-decision behavior is essential.

Furthermore, the problem of the limited driving distance of EVs is serious, particularly for long-distance driving. More precisely, EVs for long-distance driving need to recharge their batteries en route to their destination. Furthermore, EVs must stop at recharging facilities multiple times because the EVs' driving distance after quick recharging is approximately 120 km. This fact indicates that a certain number of EV-charging facilities should be deployed and all requests for recharging should be satisfied. However, Japanese EV stations currently install two rapid chargers at most. When multiple users converge at the station, a queuing (or waiting) condition is created, which may lead to the call-loss condition.

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In other words, it is beneficial to ensure that an appropriate number of chargers are installed at each station when planning EV infrastructure. Therefore, the number of vehicles entering the station must be estimated.

Thus, this research focuses on the EV-support infrastructure of charging facilities in terms of EV route-decision behavior and multiple charging stops. A mathematical model specialized for long-distance trips to estimate the number of EVs arriving at each charging facility is proposed. When considering the movements of long-distance trips, it is very important to incorporate the route of EV movements because there is not only a simple and shortest path from origin to destination, but also several alternative paths. Furthermore, it is also essential to consider an EV's multiple stops at EV-recharging facilities. An elegant mathematical model to analyze the EV-support infrastructure with respect to multiple routes and multiple-charging stops also will be discussed.

The rest of paper is organized as follows. In section "Fomulation of the Mathematical Model," a mathematical model to estimate the number of EVs arriving at each charge station is formulated, and several indices to evaluate EV movements are derived. In section "Equivalent Markov Model," the model is applied to the Japanese traffic flow and road networks. The arrival number for EV-charging stations is approximated and how users are affected by the conversion from gasoline vehicles to EVs is discussed. The conclusions of this study and several ideas for future studies are presented in section "Calculation Based on Passenger Movements in Japan."

Formulation of the Mathematical Model

Designed Network

Set up in two dimensions, the model assumes $|O|$ = trip start points (origins) $O \in (1, \ldots, o, \ldots, |O|)$; $|D|$ = trip end points (destinations) $D \in (1, \ldots, d, \ldots, |D|)$ and $|K|$ = EV charge stations $K \in (1, \ldots, k, \ldots, |K|)$. It further assumes that T_{od} EVs travel from an origin (o) to a destination (d) within the origin destination (OD) matrix. The maximum cruising distance of the EVs is L. The EVs that are unable to travel from o to d without charging their batteries must use one of several charge stations. Under these assumptions, the number of EVs arriving at each charge station will be derived.

Now, a directed graph $G = (V, E)$ connecting paths (i) through (iv) is introduced, where the distance between two points is L or less ($V = O \oplus D \oplus K$):

1. Origin (start point) → Charge station.
2. Charge station → Charge station.
3. Charge station → Destination (end point).
4. Origin (start point) → Destination (end point).

All conceivable paths of EVs traveling from the origin to destination in this network are expressed on the directed graph G, and path(s) that can be expressed on the directed graph G are the only feasible paths. An example of an EV routing network is shown in Fig. 1.

Routing Behavior

Given the previous conditions and assumptions, an EV moving from an origin (o) to destination (d) on the directed graph G is considered. Many paths linking o to d are available, including one that utilizes the shortest travel time. Understandably, the path of shortest travel time will be used most frequently, and the longer the travel time of a path, the less frequently it will be used. To incorporate this concept into the model, the probability that an EV moving from o to d will use the rth path is specified. On the basis of the multinomial logit model, the probability is defined as

$$P_r^{od} = \frac{\exp[-\gamma C_r^{od}]}{\sum_{r \in \Phi^{od}} \exp[-\gamma C_r^{od}]} \tag{1}$$

where γ = cost-decay parameter; C_r^{od} = cost of traveling the rth path from o to d; and Φ^{od} = set of paths from o to d. In this study, C_r^{od} = sum of link costs, and is expressed as

$$C_r^{od} \overset{\text{def}}{=} c_{ok_1} + \sum_{l=1}^{\Lambda-1} c_{k_l k_{l+1}} + c_{k_\Lambda d} \tag{2}$$

$$c_{ok} \overset{\text{def}}{=} \frac{d_{ok}}{v} + T \tag{3}$$

$$c_{kk'} \overset{\text{def}}{=} \frac{d_{kk'}}{v} + T \tag{4}$$

$$c_{kd} \overset{\text{def}}{=} \frac{d_{kd}}{v} \tag{5}$$

Here, the rth path is the following route from o to d: $o \to k_1 \to k_2 \to \cdots \to k_\Lambda \to d$, d_{**} is the distance between two points, v is the speed of the EV, and T is the time required for rapid battery charging. The definition of Eq. (2) assumes that EV drivers can refuel the EVs at their origin and always refuel when they reach EV station nodes. In addition, it assumes that the travel speed is constant, regardless of traffic volume.

There are a number of rationalities to introduce the logit model when describing routing behavior. First, in the field of traffic engineering, the logit model and its extensions are the main method to describe a driver's route choice. Indeed, the stochastic user equilibrium traffic assignment model internally assumes the logit model in route decision behavior (Brilon et al. 2011). Some might consider that long-distance drivers usually use major highways and have few choices in their routes. However, Japanese highway networks mainly develop in a north-south direction, so drivers tend to use other roads, which have many alternative routes. Moreover, access-egress routing behavior for highway networks is also an important factor because drivers do use highways.

How to define the alternative set Φ^{od} is now discussed. In the simple definition, the alternative set consists of simple paths. In this

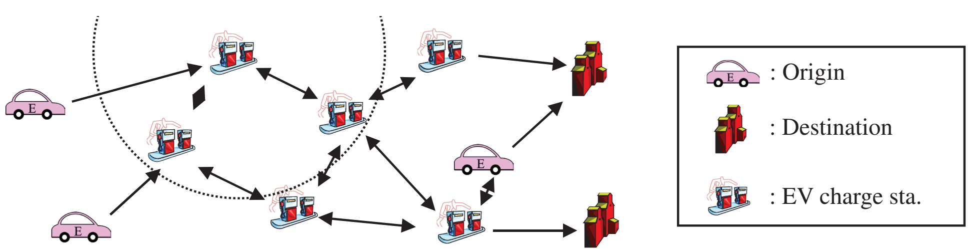

Fig. 1. Example of EV-routing network

study, however, all possible paths which connect origin and destination are considered. This strategy has the possibility of generating very long detours that would not be selected in real-world routes. Furthermore, it seems difficult to calculate the probability for large-scale networks.

On the contrary, this definition is made to enhance computational feasibility. As discussed in later sections of this paper, this definition makes it possible to perform stochastic assignment in large-scale networks.

Equivalent Markov Model

When the set of paths Φ^{od} is a subset of the entire conceivable paths, the network flow expressed by Eq. (1) is equivalent to the Markov distribution (Akamatsu 1996):

$$p_d(j|i) = \exp[-\gamma c_{ij}] \frac{W_{jd}}{W_{id}} \tag{6}$$

$$W_{id} \overset{\text{def}}{=} \sum_{r \in \Phi^{id}} \exp[-\gamma C_r^{id}] + \delta_{id} \tag{7}$$

where $p(j|i)$ = transition probability of EV traveling from node $i \in V$ to node $j \in V$; and δ_{id} = Kronecker delta symbol.

This formulation satisfies the condition

$$\sum_{j \in nbr(i)} p(j|i) = \frac{\sum_{j \in nbr(i)} \exp[-\gamma c_{ij}] W_{jd}}{W_{id}} = 1 \tag{8}$$

which should be regarded as the conditional probability. Furthermore, the equivalency between two assignments is easily confirmed by

$$\begin{aligned} P_r^{od} &= p(k_1|o) \times \prod_{l=1}^{\Lambda-1} p(k_{l+1}|k_l) \times p(d|k_\Lambda) \\ &= \exp[-\gamma c_{ok_1}] \frac{W_{k_1 d}}{W_{od}} \times \exp[-\gamma c_{k_1 k_2}] \frac{W_{k_2 d}}{W_{k_1 d}} \times \cdots \\ &\quad \times \exp[-\gamma c_{k_{\Lambda-1} k_\Lambda}] \frac{W_{k_\Lambda d}}{W_{k_{\Lambda-1} d}} \times \exp[-\gamma c_{k_\Lambda d}] \frac{W_{dd}}{W_{k_\Lambda d}} \\ &= \exp\left[-\gamma \left(c_{ok_1} + \sum_{l=1}^{\Lambda-1} c_{k_l k_{l+1}} + c_{k_\Lambda d} \right)\right] \frac{W_{dd}}{W_{od}} \\ &= \frac{\exp[-\gamma C_r^{od}]}{\sum_{k=1}^{\infty} \exp[-\gamma C_r^{od}]} \end{aligned} \tag{9}$$

The existence of Markov assignment equivalency implies that the target flow can be calculated very efficiently. More clearly, if each node uses Eq. (6) to move the next node to pass the EVs, the resulting network flow equals the assignment discussed in Eq. (1).

Note that $p(j|i)$ does not depend on the node of origin. It is confirmed that there is no subscript notation related to origin in Eq. (6). Thus, if each EV follows the Markov assignment, the movement of all EVs from arbitrary nodes spontaneously holds to Eq. (1). This fact makes it possible to calculate the many-to-one flow at once, so it is a very favorable feature of this model. Since multipath routing is used, there are multiple candidate nodes for the next step, even if the EVs are intended to go to the same destination.

How to Calculate W_{id}

To determine the value of $p(j|i)$, W_{id}, which is included in $p(j|i)$, must be calculated. The native approach is not feasible because the summation in W_{id} means considering all paths from node i to d.

Consider matrix $\mathbf{A}$ of $N \times N$, whose $[i, j]$ element is given as follows:

$$a_{ij} = \begin{cases} \exp[-\gamma c_{ij}] & \text{(if node } i \text{ to } j \text{ is connected)} \\ 0 & \text{(otherwise)} \end{cases} \tag{10}$$

The element of $\boldsymbol{A}^2$ yields

$$a_{ij}^{[2]} = \sum_{k=1}^{N} a_{ik} a_{kj} = \sum_{\{k|(i,k),(k,j) \in E\}} \exp[-\gamma(c_{ik} + c_{kj})] = \sum_{r \in \Phi_2^{ij}} \exp[-\gamma C_r^{ij}] \tag{11}$$

where Φ_L^{ij} = set of paths that connect nodes i and j in L steps. Similarly, the typical element of $\boldsymbol{A}^L$ is given by

$$a_{ij}^{[L]} = \sum_{r \in \Phi_L} \exp[-\gamma C_r^{ij}] \tag{12}$$

From the definition

$$\Phi^{ij} = \Phi_1^{ij} \oplus \Phi_2^{ij} \oplus \cdots \tag{13}$$

is derived. Consequently, the value of W_{id} can be obtained as follows ($\boldsymbol{I}$ is the identity matrix):

$$\boldsymbol{W} = \boldsymbol{I} + \boldsymbol{A} + \boldsymbol{A}^2 + \boldsymbol{A}^3 + \cdots \tag{14}$$

where the element of matrix $\boldsymbol{W}$ is

$$W_{ij} = \sum_{r \in \Phi^{ij}} \exp[-\gamma C_r^{ij}] + \delta_{id} \tag{15}$$

Furthermore, if matrix $\boldsymbol{A}$ satisfies the Hawkins-Simon condition (Hawkins and Simon 1949; Seneta 2006), the following formula is obtained:

$$[\boldsymbol{I} - \boldsymbol{A}]^{-1} = \boldsymbol{I} + \boldsymbol{A} + \boldsymbol{A}^2 + \boldsymbol{A}^3 + \cdots \tag{16}$$

Therefore, the value of W_{ij} is obtained as follows:

$$\boldsymbol{W} = [\boldsymbol{I} - \boldsymbol{A}]^{-1} \tag{17}$$

As indicated previously, determining the link-weighted matrix $\boldsymbol{A}$ yields the transition probability $p(j|i)$ in a straightforward manner. This means that all EV movements will be calculated only by setting matrix $\boldsymbol{A}$. This is feasible because Eq. (16) is just one calculation for the matrix inverse.

Derivation of Arrival Number to EV Station

Note that the EV travel can be described by the Markov model. Next, the Markov process that reaches destination (d) is investigated. When the transition matrix between nodes in the Markov process is defined by Eq. (18), the number (N_{id}) of EVs entering each node (while traveling toward the destination) is determined by Eq. (19):

$$P_d \overset{\text{def}}{=} [p_d(j|i)] \tag{18}$$

$$\begin{pmatrix} N_{1d} \\ N_{2d} \\ \vdots \\ N_{|V|d} \end{pmatrix}^T = \begin{pmatrix} T_{1d} \\ T_{2d} \\ \vdots \\ T_{|V|d} \end{pmatrix}^T [I - P_d]^{-1} \qquad (19)$$

where T_{od} = number of EV movements from origin (o) to destination (d). Thus, the number (N_k) of EVs arriving at charge station k can be derived as follows (independent of destination):

$$N_k = \sum_{d \in D} N_{kd} \qquad (20)$$

This is one of the most important pieces of information to calculate in this paper.

Average Cost from Origin to Destination

At the end of the formulation, the average cost from origin (o) to destination (d) is also calculated.

Now, focusing on the movement from origin (o) to destination (d), the probability of pass-through link (i, j) is described as q_{ij}^{od}. Through the formula stated previously, the average cost $\langle C_{od} \rangle$ can be written as

$$\begin{aligned} \langle C_{od} \rangle &= \sum_{r \in \Phi^{od}} \cdot P_r^{od} \cdot C_r^{od} \\ &= \sum_{(i,j) \in E} q_{ij}^{od} \cdot c_{ij} \end{aligned} \qquad (21)$$

In order to calculate Eq. (21), it is necessary to derive q_{ij}^{od}. Here, focusing on the fact that EV movements can be described as a Markov model, the Markov process to destination node d is considered again. Then, for the movement from origin (o) to destination (d), the probability of passing each node Q_i^{od} is calculated as

$$\begin{pmatrix} Q_1^{od} \\ Q_2^{od} \\ \vdots \\ Q_{|N|}^{od} \end{pmatrix}^T = \boldsymbol{e}_o^T [I - P_d]^{-1} \qquad (22)$$

where $\boldsymbol{e}_o^T$ = unit vector in which only the oth element is 1 and 0 otherwise. Consequently, the probability q_{ij}^{od} that passes through link (i, j) is

$$[q_{ij}^{od}] = \mathrm{diag}(Q_1^{od}, Q_2^{od}, \cdots, Q_{|N|}^{od}) P_d \qquad (23)$$

where $\mathrm{diag}(\cdot)$ = diagonal matrix with the applicable component as a diagonal element.

Eq. (23) is basically the same as the Akamatsu model (Akamatsu 1996), and thus, there are minor contributions to theoretical aspect. Rather, the key finding of this paper is to show that the Akamatsu model can clearly apply to EV routing behavior and the arrangement of mathematical expressions for EV routing, such as average travel cost calculation. Thus, it is also very important to show that the formulation is applicable for practical use. For this purpose, the quite large-scale problem with realistic data is calculated in the next section.

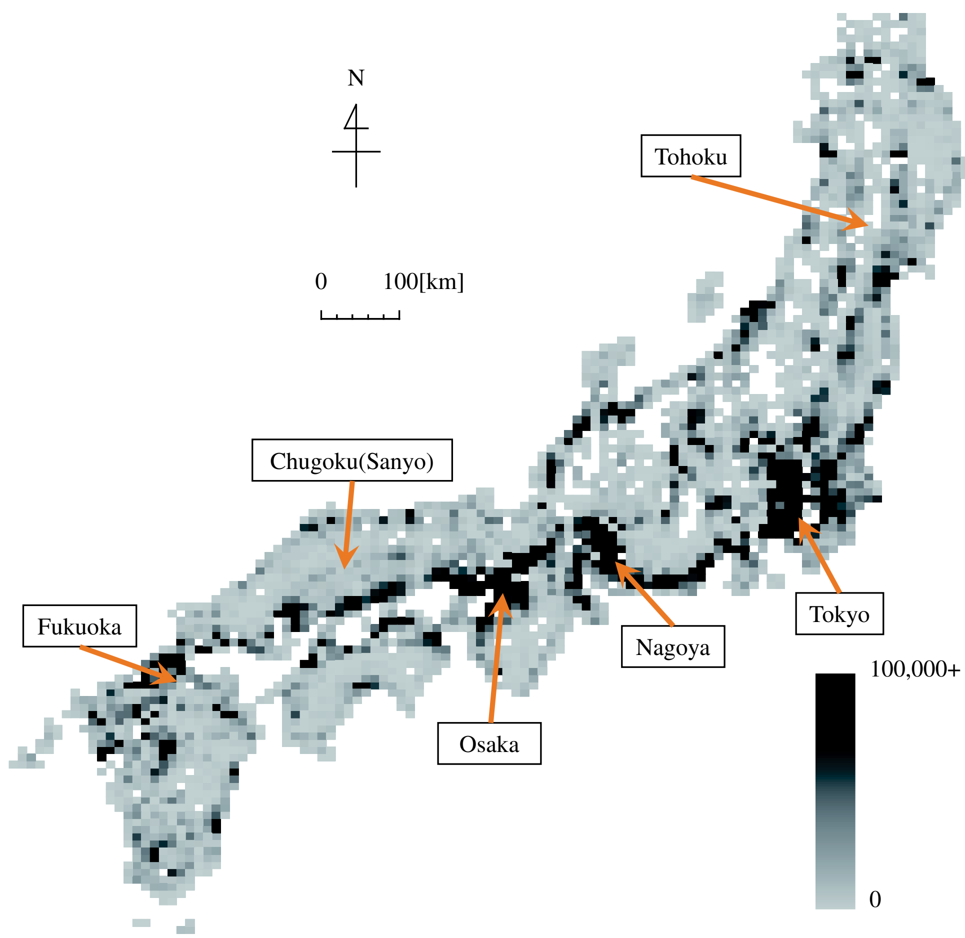

Fig. 2. Mesh population data

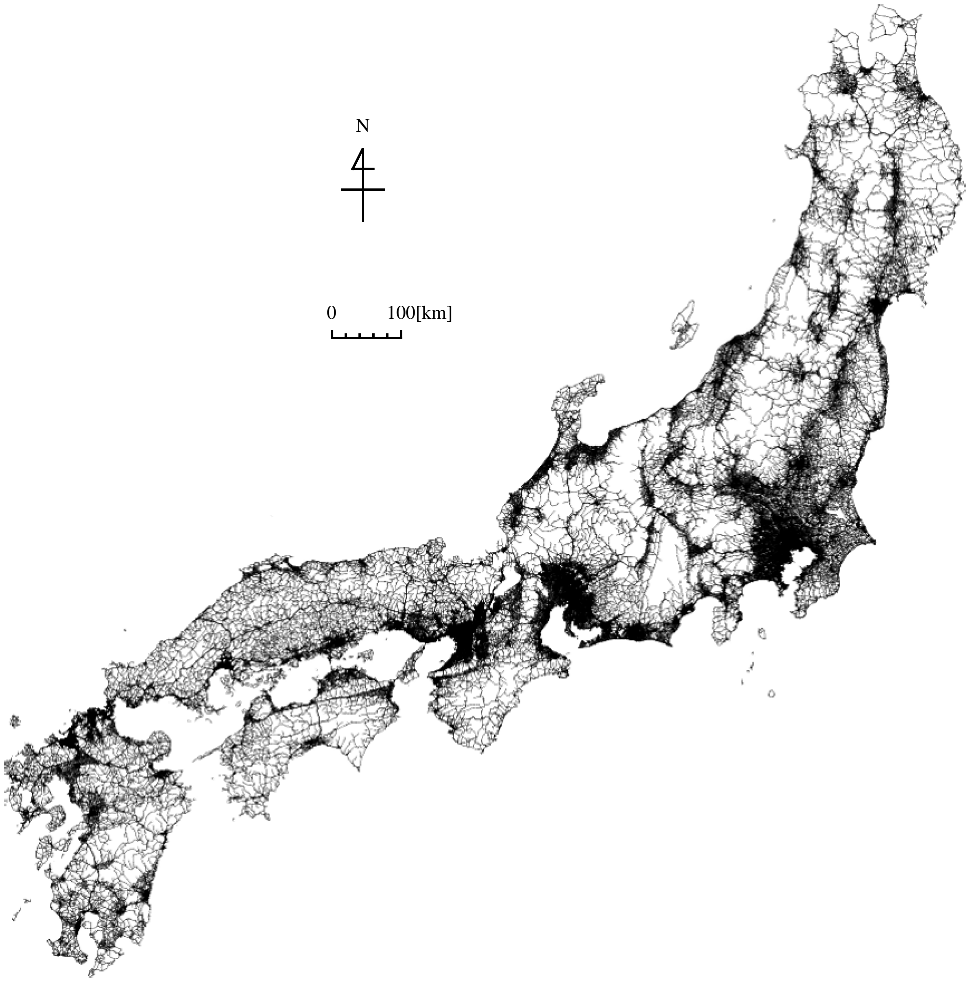

Fig. 3. Detailed road network data

Calculation Based on Passenger Movements in Japan

Preparation of OD Data and Network

The model described in the previous sections is applied to the movement of traveling passengers in Japan (excluding Hokkaido and Okinawa) to estimate the number of EVs arriving at the charge station. In the following calculation, it is assumed that the vehicles driving in the target region were all replaced with EVs.

First, how to prepare OD data is explained. From the Third Interregional Travel Survey (MLIT 2010b) issued by the Japanese Ministry of Land, Infrastructure, Transport, and Tourism, there are several OD flow data that focus o,nvarious transportation methods and trip purposes. In this study, the OD data of "vehicle," "weekday," and "all purpose" is extracted. This OD data is aggregated between 207 zone movements. Thus, the 10-km mesh population data of national population census 2010 (MLIT 2010a) is also used, and OD data is proportionally allocated to each mesh. Using the previous calculation, the OD data between 3,460 mesh movements is created. The centers of each mesh are set as origin and destination points. Mesh population data is shown in Fig. 2.

Table 1. Size of Detailed Road Network Data

Item	Size
Node	1,269,703
Link	1,682,550
Total distance (km)	432,405

Next, the EV charge-station data is prepared. According to CHAdeMO Association, 2,151 EV-charging stations have rapid chargers currently installed (CHAdeMO 2013). The locations of these 2,151 EV stations are used as the distribution.

Finally, the travel cost between two arbitrary points based on the real road network is calculated. In this study, the detailed road network data, which are also used for car navigation systems (Sumitomo 2005), is prepared, and the shortest distance path is calculated. Furthermore, the distance is converted to travel time (= cost) under the assumption that EV travel speed $v = 80\ \mathrm{km/h}$ on highways, $v = 60\ \mathrm{km/h}$ on metropolitan expressways, and $v = 30\ \mathrm{km/h}$ on ordinary roads. Fig. 3 shows the detailed road network data, and Table 1 is the size of this data.

Using this geographic information science (GIS) data, the EV routing network, which was explained in the formulation, is constructed. Fig. 4 shows the EV routing network in this study, and Table 2 shows the size of this network. It is assumed that the time for rapid battery charging (T) is 40 min and the maximum cruising distance of the EV (L) is 120 km.

Estimated Number of EV Arrival

The number of EV arrival to each station is first analyzed based on the mathematical model. It is assumed that parameter γ is 0.1.

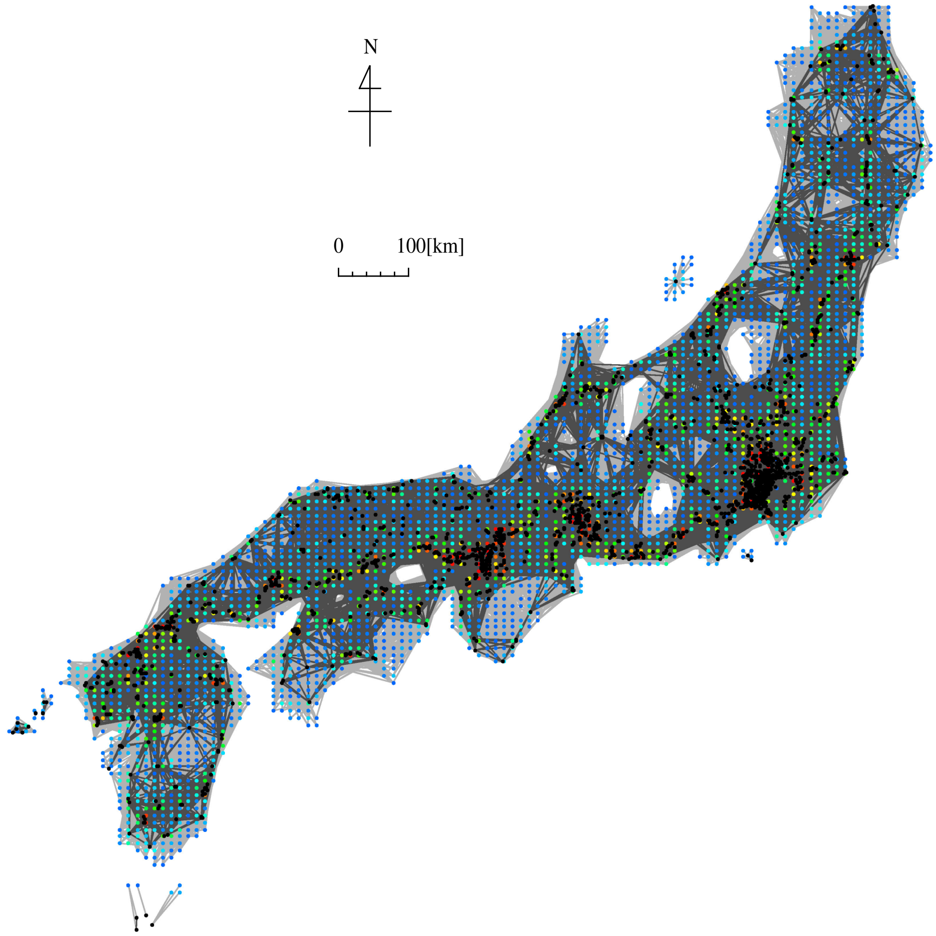

Fig. 4. EV-routing network

Table 2. Size of EV Routing Network Data

Item	Size
OD destination	3,460
EV-charging station	2,151
Node	10,715
Link	2,700,048

Based on the previous network, the calculation results are shown in Fig. 5 (indicating the number of EVs arriving at the charge station per hour).

From the figure, it can be confirmed that the stations located around large cities and near arterial roads are the most frequently used. It can be seen precisely that the number of EVs arriving at stations near metropolitan areas, such as Tokyo, Nagoya, Osaka, and Fukuoka, evidently increases. In addition, the regions connecting Tokyo, Nagoya, and Osaka, known as the Higashi-Meihan regions, have been confirmed to have a significant number of arriving EVs. The previously mentioned results can be properly understood because the number of arriving EVs must be proportional, to some extent, to the passing traffic at that spot from a general perspective. Meanwhile, rural areas such as Tohoku, Chugoku, and Sanyo do not have as many EVs as metropolitan areas. Generally, these rural areas have a number of stations, where approximately 100 EVs are estimated to use. Therefore, a certain amount of EV stations must be installed even in rural areas, which are low-demand areas. From this viewpoint, it is suggested that infrastructure improvements are important on a nationwide scale. The most important point when planning the strategy of infrastructure improvements is appropriate allocation based on EV arrivals.

Fig. 6 is a histogram of the number of EVs arriving at the charge station per hour. Approximately 72.6% of the 2,151 stations are visited by 100 EVs per hour or less (average number of EV visits is 95.7 cars/h). In contrast, the number of EVs arriving at the busiest station is 3,793 cars/h, suggesting that many chargers will be needed in that station.

The queuing theory (Haviv 2013) is introduced here. An EV-charging station can follow the typical M/M/s queuing model. Therefore, $\lambda/s\mu < 1$ must be satisfied; here, λ = arrival rate of EV per hour; μ = service rate of EV quick charger per hour; and s = number of EV quick chargers in an EV-charging station. This equation means that the number of arrivals to the queuing system cannot exceed the total service potential of the system. Therefore, this equation must be satisfied in any queuing model. Now, it is assumed that each EV occupies the charging station for approximately 40 min on average (considered storing, charging, and retrieving) and the number of EV chargers is two (most stations only have one charger at present), and the actual diffusion rate of EVs is α. Then

$$\frac{3{,}793 \times \alpha}{2 \times 60/40} < 1 \tag{24}$$

$$\alpha < \frac{2 \times 60/40}{3{,}793} \approx 0.079\% \tag{25}$$

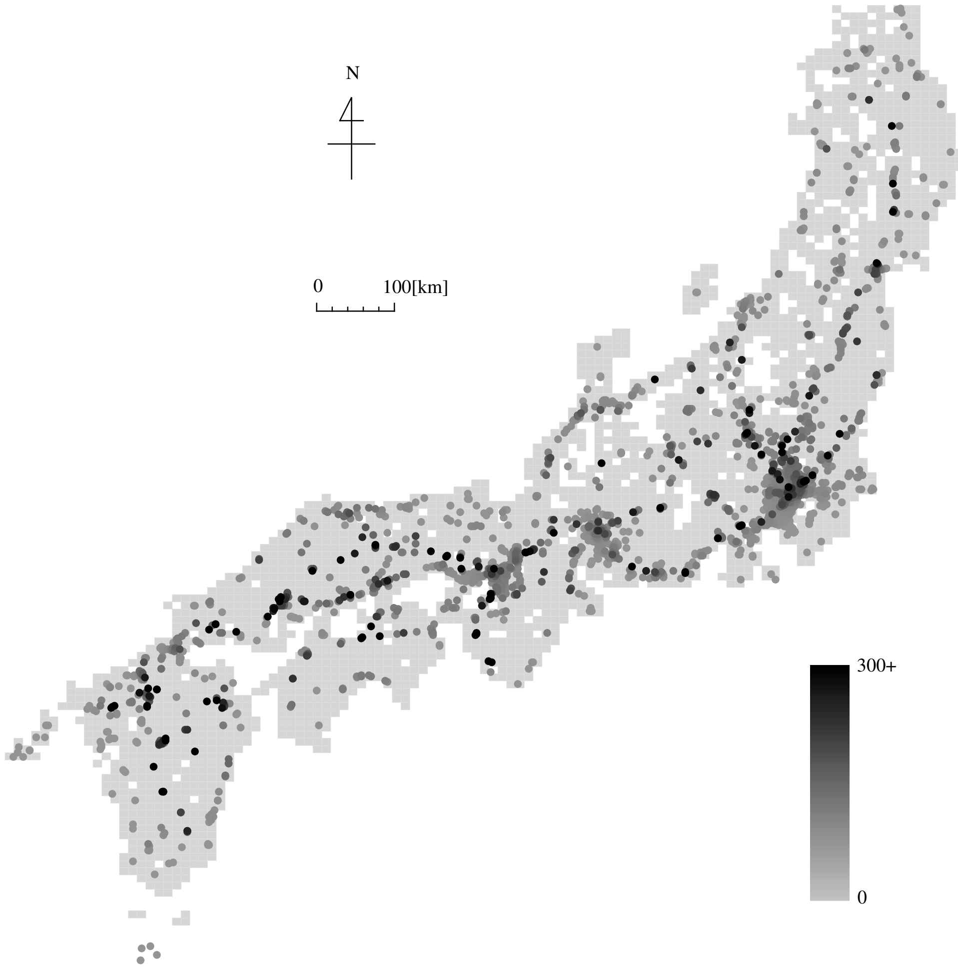

Fig. 5. Number of EV arrivals per hour

are derived. Thus, in the actual situation, this can be managed if the actual diffusion rate of EVs is less than 0.079%.

However, the condition $\lambda/s\mu < 1$ only assures that the waiting queue would not increase infinitely. In a practical sense, queue length (and waiting time) infinitely diverges when α is close to the limit stated in Eq. (25). Fortunately, by the formula of $M/M/s$ queuing model, the waiting time at EV stations can be also calculated. For example, to set the average waiting time to less than 5 min, $\alpha < 0.026\%$ should be satisfied from M/M/s formula. In any case, the most important point is to calculate the arrival number of EVs, which is shown in Fig. 5.

Comparison of Average Time

Next the average travel time from origin to destination is examined. It is also a very important factor to evaluate the convenience for EV users. The average travel time by EVs can be derived as well, using the previously mentioned formulas.

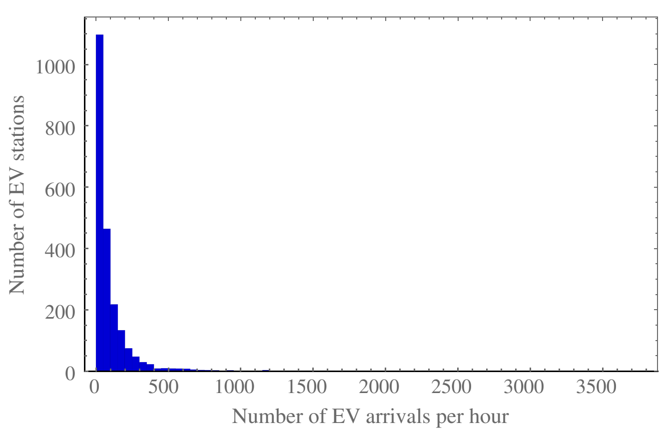

Fig. 6. Histogram of EV arrivals per hour

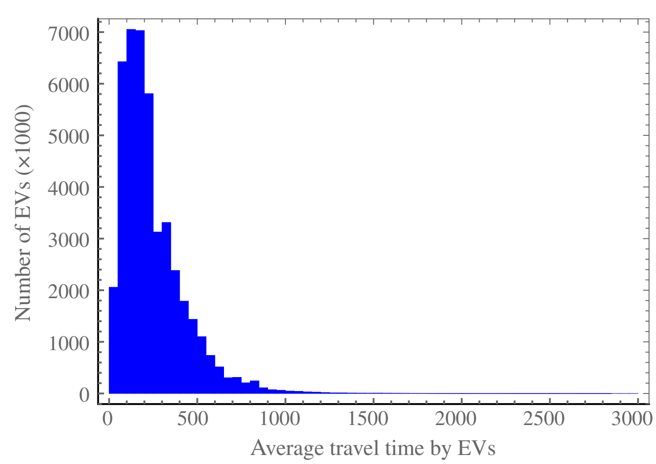

Fig. 7. Histogram of the average travel time by EVs

Fig. 7 is a histogram of the average travel time (including charging time) on an EV routing network. Approximately 63% of EVs travel 6 h or less (average number of EV visits 242.15/min). In contrast, the maximum travel time is near 50 h.

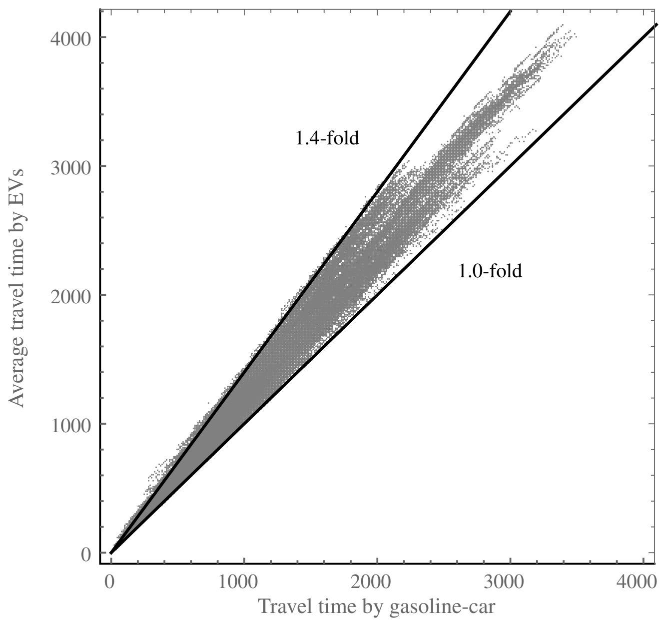

Fig. 8. Scatter diagram of the EV time and the gasoline car time

By calculating the time of shortest distance, which supposes gasoline cars, both times are now compared to evaluate the inconvenience of EV usage. Fig. 8 is a scatter diagram of the EV time and the gasoline car time. From the figure, it can be confirmed that EV users are obligated at a maximum 1.4-fold time. This is mainly because of the need for detouring to charge the battery.

To illustrate the regional difference in terms of EV trips, the following index is calculated:

$$R_o = \sum_d \frac{\langle C_{od} \rangle}{C_{od}^{\min}} \tag{26}$$

where $\langle C_{od} \rangle$ = average travel time by EVs; and $C_{od}^{\min}$ = time of the shortest distance by gasoline cars. As is clear from the definition, R_o = average ratio, which indicates the proportion of EV and gasoline cars. If R_o is large, the region would be disadvantaged by the conversion from gasoline car to EV. The calculation result of R_o is shown in Fig. 9. From the figure, it can be confirmed that the Kyushu regions, and Ibaraki prefecture have large R_o which means the "EV refugee." Therefore, in terms of the development of EV infrastructure, the Kyushu region and Ibaraki prefecture are very important for investment. In parallel, controlling the diffusion rate of EVs may be another important way to cope with the lack of infrastructure in such areas.

Note that the result R_o is not weighted by OD data and population. Obviously, it is easy to incorporate the weighted average of $\langle C_{od} \rangle / C_{od}^{\min}$ by OD data. However, if the weighted average were introduced, it would be apparent that the ratio of rural areas tends to be worse than that of metropolitan areas. This is because

Fig. 9. Average ratio of travel time by EVs and by gasoline cars

$\langle C_{od}\rangle / C_{od}^{\min}$ generally increases with the increasing distance between origin and destination. Such analysis is not the authors' intention. By not taking account of OD data and populations, it is possible to illustrate the inconvenience caused by the location of EV stations.

Conclusion

This research focused on the EV support infrastructure of charging stations and proposes a mathematical model to estimate the number of EVs arriving at each EV-charging station. In particular, a mathematical model to estimate the number of EVs arriving at each charging station with respect to multiple routes and multiple stops is formulated, and basic properties of the previous problem are discussed. Furthermore, the model is applied to Japanese traffic flows and the number of EV arrivals to EV-charging stations is calculated. In addition, by comparing EV travel time with that of gasoline-fueled cars, the priority regions for investment of EV support infrastructure are illustrated. Future work should include analyzing more practical situations with several collaborated facilities as well as social optimal system, which maximizes the user benefits.

Appendix. Example of Routing Calculation

To promote understanding, a brief example of EV routing is provided. The network consisting of four nodes and five links described in Fig. 10 is considered. For simplicity, all links are assumed to have unit cost (i.e., $c_{**} = 1$). The goal is to move EVs from node 1 to node 4.

As is easily confirmed, there are three candidate paths from node 1 to node 4:

- Part I 1→2→4
- Part II 1→3→4
- Part III 1→2→3→4

The total costs of paths I and II are $C_{\mathrm{I}} = C_{\mathrm{II}} = 2$, and the cost of path III is $C_{\mathrm{III}} = 3$. Thus, from Eq. (1), the probability that each path is used is derived as follows:

$$\textbf{path I} \quad \frac{\exp[-\gamma C_{\mathrm{I}}]}{\sum_{r=\mathrm{I}}^{\mathrm{III}} \exp[-\gamma C_r]} = \frac{x^2}{x^3 + 2x^2} \tag{27}$$

$$\textbf{path II} \quad \frac{\exp[-\gamma C_{\mathrm{II}}]}{\sum_{r=\mathrm{I}}^{\mathrm{III}} \exp[-\gamma C_r]} = \frac{x^2}{x^3 + 2x^2} \tag{28}$$

$$\textbf{path III} \quad \frac{\exp[-\gamma C_{\mathrm{III}}]}{\sum_{r=\mathrm{I}}^{\mathrm{III}} \exp[-\gamma C_r]} = \frac{x^3}{x^3 + 2x^2} \tag{29}$$

where $x = \exp[-\gamma]$.

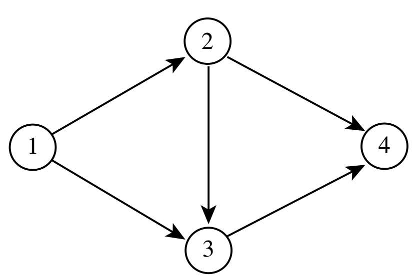

Fig. 10. Sample network

As discussed previously, the transition probability from node i to node j is calculated. From Eq. (10), matrix $\boldsymbol{A}$ for this network is

$$\boldsymbol{A} = \begin{bmatrix} 0 & x & x & 0 \\ 0 & 0 & x & x \\ 0 & 0 & 0 & x \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{30}$$

and thus, $\boldsymbol{W}$ is calculated from Eq. (17) as follows:

$$\boldsymbol{W} = \begin{bmatrix} 1 & x & x^2 + x & x^3 + 2x^2 \\ 0 & 1 & x & x^2 + x \\ 0 & 0 & 1 & x \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{31}$$

Then, from Eq. (6), it is obtained as follows:

$$\begin{aligned} [p(2|1), p(3|1)] &= \left[\exp(-\gamma c_{12})\frac{W_{24}}{W_{14}}, \exp(-\gamma c_{13})\frac{W_{34}}{W_{14}}\right] \\ &= \left\{x \times \frac{x^2 + x}{x^3 + 2x^2}, x \times \frac{x}{x^3 + 2x^2}\right\}, \end{aligned} \tag{32}$$

$$[p(3|2), p(4|2)] = \left(x \times \frac{x}{x^2 + x}, x \times \frac{1}{x^2 + x}\right) \tag{33}$$

$$[p(4|3)] = \left(x \times \frac{1}{x}\right) \tag{34}$$

It is certain that the Markov assignment described by Eqs. (32)–(34) is equivalent to Eqs. (27)–(29); for example,

$$\textbf{path I} \quad p(2|1) \times p(4|2) = \frac{x^2}{x^3 + 2x^2} \tag{35}$$

This is a brief example of EV routing.

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